Algebra III Section 1.5

Writing Equations of Parallel and Perpendicular Lines

Parallel Lines - Two nonvertical lines in a plane are parallel if and only if their slopes are equal and they have no points in common. Two vertical lines are always parallel.

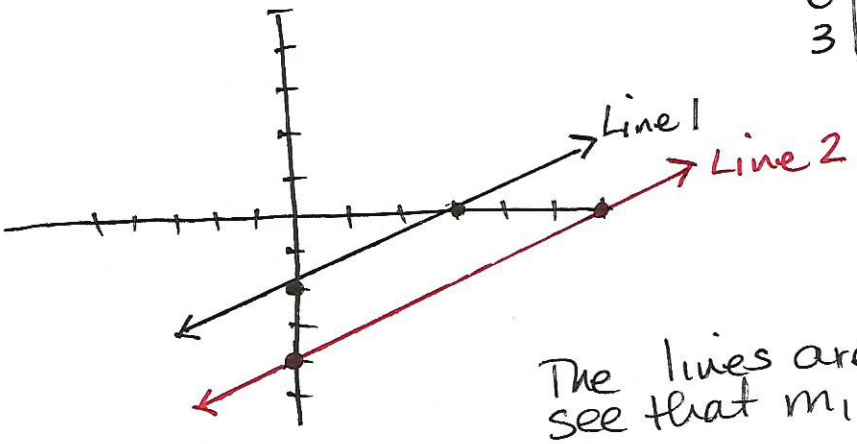
Given: Line 1 $2x - 3y = 6$ $m_1 = -\frac{2}{-3} = \frac{2}{3}$

Line 2 $2x - 3y = 12$ $m_2 = -\frac{2}{-3} = \frac{2}{3}$

When we graph each of these lines, we see that they are parallel

Line 1	
x	y
0	-2
3	0

Line 2	
x	y
0	-4
6	0



The lines are parallel and we see that $m_1 = m_2$.

Conclusion: If two lines are parallel, their slopes are =.

Perpendicular Lines- Two nonvertical lines in a plane are perpendicular if and only if their slopes are opposite reciprocals. $m_1 \cdot m_2 = -1$

Given Line 1 $2x - 3y = 6$ $m_1 = -\frac{2}{-3} = \frac{2}{3}$

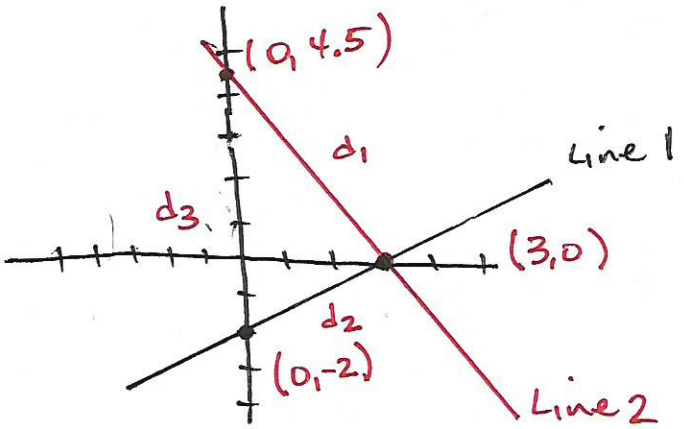
Line 2 $3x + 2y = 9$ $m_2 = -\frac{3}{2} = -\frac{3}{2}$

Line 1

x	y
0	-2
3	0

Line 2

x	y
3	0
0	4.5



Note that we form a triangle from our points. We will use the Pythagorean Theorem and the distance formula to show whether we have a right triangle or not. If we have a right triangle, then we have perpendicular lines.

$d_1 = \sqrt{(3-0)^2 + (0-4.5)^2} = \sqrt{3^2 + (4.5)^2} = \sqrt{29.25}$

$d_2 = \sqrt{(3-0)^2 + (0-(-2))^2} = \sqrt{3^2 + 2^2} = \sqrt{13} =$

$d_3 = \sqrt{(0-0)^2 + (-2-4.5)^2} = \sqrt{-(6.5)^2} = \sqrt{42.25}$

$d_3^2 = d_1^2 + d_2^2$

$\sqrt{42.25} = \sqrt{13 + 29.25}$

$42.25 = 42.25$

The Theorem works which indicates the presence of a right angle. This means we have perpendicular lines.

$m_1 \cdot m_2 = -1$

$\frac{2}{3} \cdot -\frac{3}{2} = -\frac{6}{6} = -1$

If we know a second line is parallel or perpendicular to a first line, this gives a clue about the slope of the second line. If we also know a point found on the second line, then we are able to use the Point-Slope Method to solve for the equation of the second Line.

Ex. Line 2 is parallel to Line 1 $-4x-9y=-23$ and contains the point $(12,-15)$.

Slope of Line 1 $= m = -\frac{4}{-9} = \frac{4}{9}$, therefore $m_2 = \frac{4}{9}$

Point-Slope Method $y-y_1 = m(x-x_1)$

$$y-(-15) = \frac{4}{9}(x-12)$$

$$y+15 = \frac{4}{9}x - \frac{48}{9}$$

$$9\left(y+15 = \frac{4}{9}x - \frac{48}{9}\right)$$

$$9y+135 = 4x-48$$

$$135+48 = 4x+9y$$

$$183 = 4x+9y$$

$$\underline{4x+9y=183}$$

Ex Line 2 is perpendicular to line 1 → $4x-8y=24$ and contains the point $(2,-2)$.

$m_1 = -\frac{4}{-8} = \frac{4}{8} = \frac{1}{2}$

$m_2 = \frac{-2}{1}$ use slope 2 and the given point to solve for line 2.

$y-(-2) = -2(x-2)$

$y+2 = -2x+4$

$+2x+y = 4-2$

$2x+y = 2$

Line 2 is $2x+y=2$

page 36, #28 line 1 = $5y-4x=10$ included point $(-15, 8)$

a) Find line 2 parallel to line 1

b) Find line 2 perpendicular to line 2

* Be careful- $m = -\frac{A}{B}$; A is the x coefficient, B is the y coefficient

* $m_1 = -\frac{-4}{5} = \frac{4}{5}$

a) $m_2 = 4/5$ $(-15,8)$ $y-8 = \frac{4}{5}(x-(-15))$

$y-8 = \frac{4}{5}(x+15)$ → $y-8 = \frac{4}{5}x+12$

$5(y-8) = 4x+60$ → $5y-40 = 4x+60$

$-100 = 4x-5y$; $\boxed{4x-5y=-100}$

b) $m_2 = -\frac{5}{4}$ $(-15,8)$ $y-8 = -\frac{5}{4}(x-(-15))$

$y-8 = -\frac{5}{4}x - \frac{75}{4}$ → $44-32 = -5x-75$

$5x+4y = -75+32$

$\boxed{5x+4y=-43}$