Section 1.2 Composition of Functions

operations of Functions

Sum	$f(x)+g(x) = (f+g)x$
Difference	$f(x)-g(x) = (f-g)x$
Product	$f(x)\cdot g(x) = (f\cdot g)x$
Quotient	$\frac{f(x)}{g(x)}, g(x) \neq 0 = \left(\frac{f}{g}\right)x$

This allows the combining of functions followed by the evaluation of the combined function at the given value.

Ex $f(x) = x+2$
$g(x) = x-1$

Solve for $((f+g)(x))$ and evaluate at $x=3$

$f(x)+g(x) = x+2+x-1 = 2x+1$
$(f+g)(3) = 2(3)+1 = 6+1 = 7$
$f(3)+g(3) = (3+2)+(3-1) =$
$5 + 2 = 7$

Solve for $(f-g)(x)$ and evaluate at $x=3$

$f(x)-g(x) = (x+2)-(x-1)$
$= x+2-x+1 = 3$
$f(3)-g(3) = (3+2)-(3-1)$
$= 5-2 = 3$

$f(x) = x+2$
$g(x) = x-1$

Solve for $(f \cdot g)(x)$ and evaluate at $x=3$

$$(f \cdot g)(x) = f(x) \cdot g(x)$$
$$= (x+2)(x-1) = x^2 + x - 2$$
$$(f \cdot g)(3) = 3^2 + 3 - 2 = 10$$
$$f(3) \cdot g(3) = (3+2) \cdot (3-1)$$
$$= 5 \cdot 2 = 10$$

Solve for $\left(\frac{f}{g}\right)(x)$ and evaluate at $x=3$

$$\left(\frac{f}{g}\right)(x) = \frac{x+2}{x-1}$$
$$\left(\frac{f}{g}\right)(3) = \frac{3+2}{3-1} = \frac{5}{2}$$

For each new function, the domain consists of those values of x common to the domain of f and g. The domain of the quotient function is further restricted by excluding any values that make the denominator, g(x), zero.

Composition of Functions

We take and evaluate a function by inserting into it the value of the second function.

Ex. $f(x) = x^2 + 2x + 1$

$g(x) = 3a$

The composite of $f(x) \circ g(x)$ or stated as $f(g(x))$ is the f function evaluated at 3a (the g function).

$$f(x) \circ g(x) = f(g(x)) = f(3a) = (3a)^2 + 2(3a) + 1$$
$$= 9a^2 + 6a + 1$$

Ex. $f(x) = 3x^2 - 2x + 5$

$g(x) = x - 1$

$$f(x) \circ g(x) = f(g(x)) = f(x-1) = 3(x-1)^2 - 2(x-1) + 5$$
$$= 3(x^2 - 2x + 1) - 2(x-1) + 5$$
$$= 3x^2 - 6x + 3 - 2x + 2 + 5$$
$$= 3x^2 - 8x + 10$$

We can also reverse the composites

$$g(x) \circ f(x) = g(f(x)) = g(3x^2 - 2x + 5)$$
$$= (3x^2 - 2x + 5) - 1$$
$$= 3x^2 - 2x + 4$$

Notice that the composites do not result in the same answer.

To Solve for the domain of a composite function we follow the steps given below.

Ex. $f(x) = \sqrt{x+2}$ $\quad g(x) = \frac{1}{x+1}$

a) find the domain of f(x) and g(x) separately

$f(x) = \sqrt{x+2}$ Avoid (- numbers in squareroot)

$x+2 \geq 0$; $x \geq -2$

$g(x)$ $\frac{1}{x+1}$ Avoid a value of 0 in the denominator

$x+1 \neq 0$; $x \neq -1$

b) To find the domain of $f(x) \circ g(x)$ we say first that the domain of g(x) is included $x \neq -1$.

Secondly, we pair the function g(x) with the domain of f to find any other limitations.

$$\underbrace{\frac{1}{x+1}}_{g(x)} \geq \underbrace{-2}_{\text{domain of } f}$$

c) Solve for x:

$$1 \geq -2(x+1)$$
$$1 \geq -2x-2$$
$$-2x-2 \leq 1$$
$$-2x \leq 1+2$$
$$-2x \leq 3$$
$$x \geq \frac{-3}{2}$$

Combining domains we say the domain of $(f \circ g)(x)$ is $\{x : x \neq -1, x \geq -3/2\}$

$(f \circ g)(x) = \sqrt{\frac{1}{x+1} + 2}$

a) first $\frac{1}{x+1} \neq \frac{1}{0}$

so $x \neq -1$

b) second $\frac{1}{x+1} + 2 \geq 0$

$\frac{1}{x+1} \geq -2$

$\frac{x+1}{1} \geq \frac{1}{-2}$

$x \geq -\frac{1}{2} - 1$

$x \geq -1\frac{1}{2}$ or $x \geq -\frac{3}{2}$

this supports our conclusion about the domain of $f \circ g\,(x)$.

Iteration - The process of taking the composite of successive values in the same function.

Ex. $f(x) = x^2 - 1 \qquad x_0 = 2$

$f(2) = (2)^2 - 1 = 4 - 1 = 3 \qquad x_1 = 3$

$f(3) = (3)^2 - 1 = 9 - 1 = 8 \qquad x_2 = 8$

$f(8) = (8)^2 - 1 = 64 - 1 = 63 \qquad x_3 = 63$

x_1, x_2, x_3 are the first 3 iterates of the function $f(x) = x^2 - 1$ when $x_0 = 2$.