

Trig/Precalculus
Midterm Exam Review
Mr. Roy

Name Key
 Date _____
 Period _____

Square root functions

1. $\sqrt{x+10} = 5 - \sqrt{3-x}$

$x = -6, -1$

$(\sqrt{x+10})^2 = (5 - \sqrt{3-x})^2$

$x+10 = 25 - 10\sqrt{3-x} + 3-x$

$(-x+9)^2 = (5\sqrt{3-x})^2$

$(x+6)(x+1) = 0$

$2x - 18 = -10\sqrt{3-x}$

$x^2 - 18x + 81 = 25(3-x)$

$x = -6, -1$

$-x + 9 = 5\sqrt{3-x}$

$x^2 - 18x + 81 - 75 + 25x = 0$

$x^2 + 7x + 6 = 0$

$x \geq -5/4 \quad x \leq 95/4$

2. $\sqrt{4x+5} \leq 10$

$(\sqrt{4x+5})^2 \leq 10^2$

$4x \leq 95$

$4x + 5 \leq 100$

$x \leq 95/4 \quad \checkmark$

$4x + 5 = 0$

$4x = -5$

$x = -5/4$

Zeros for a polynomial

3. Given that one zero is 1, find all zeros of $f(x) = 3x^3 + 4x^2 - 5x - 2$

$$\begin{array}{r|rrrr} & 3 & 4 & -5 & -2 \\ \hline 1 & 3 & 7 & 2 & \\ \hline & 3 & 7 & 2 & 0 \end{array}$$

$x = 1, -1/3, -2$

$3x^2 + 7x + 2 = 0$

$(3x+1)(x+2) = 0$

$x = -1/3, -2$

$x = -2, 5, 3$

4. Given that one zero is -2, find all zeros of $f(x) = x^3 - 6x^2 - x + 30$

$$\begin{array}{r|rrrr} & 1 & -6 & -1 & 30 \\ \hline -2 & & -2 & 16 & -30 \\ \hline & 1 & -8 & 15 & 0 \end{array}$$

$x^2 - 8x + 15 = 0$

$(x-5)(x-3) = 0$

$x = 5, 3$

Quadratic Formula & Discriminant

5. Find the discriminant and describe the nature of the roots of $4x^2 - 6x + 3 = 0$

$$b^2 - 4ac = (-6)^2 - 4(4)(3) = 36 - 48 = -12$$

$$D = -12$$

Discriminant = 2 imaginary roots

Nature of the roots = _____

6. Using the quadratic formula, determine the roots for $P(x) = 4w^2 + 19w - 5 = 0$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-19 \pm \sqrt{19^2 - 4(4)(-5)}}{8} \quad w = \frac{1}{4}, -5$$

$$x = \frac{-19 \pm \sqrt{441}}{8} = \frac{-19 \pm 21}{8} = \frac{2}{8}, \frac{-40}{8} = \frac{1}{4}, -5$$

Odd & Even Functions

7. Determine whether the function $f(x) = x^3 + 2$ is odd, even, or neither. 7. Neither

Prove your answer.

odd $f(-x) = -f(x)$ Let $x = -2$

$$f(-(-2)) = f(2) = 2^3 + 2 = 10 \quad 10 \neq 6$$

$$-f(-2) = -(-2^3 + 2) = -(-8 + 2) = 6$$

even When $x = -2$, $f(x) = -6$
Does $(2, -6)$ work?
 $x = 2$, $f(x) = 10$ No

8. Determine whether the function $f(x) = x^2 - 2$ is odd, even, or neither. 8. even

Prove your answer.

odd Let $x = -2$ No

Does $f(-x) = -f(x)$? yes

$$f(-x) = f(-(-2)) = 2^2 - 2 = 2$$

$$-f(x) = -(f(-2)) = -((-2)^2 - 2) = -2$$

$x = -2$ $f(x) = 2$
Does $(2, 2)$ work? even
 $2^2 - 2 = 4 - 2 = 2$ yes

End Behavior

9. Describe the end behavior of $y = -x^4 + x^2$.

9. $x \rightarrow +\infty$ $f(x) \rightarrow -\infty$
 $x \rightarrow -\infty$ $f(x) \rightarrow -\infty$

10. Describe the end behavior of $y = -x^3 + 2x^2 - 3x + 5$

10. $x \rightarrow +\infty$ $f(x) \rightarrow -\infty$
 $x \rightarrow -\infty$ $f(x) \rightarrow +\infty$

Descartes Rule of Signs/Upper & Lower Bound/Possible Rational Roots

11. For $f(x) = 4x^3 + x^2 - 11x + 3$

$f(-x) = -4x^3 + x^2 + 11x + 3$

Determine the following:

- a) Number of roots = 3
- b) Possible rational roots $\pm \frac{3}{4} \pm \frac{3}{2} \pm 3 \pm \frac{1}{4} \pm \frac{1}{2} \pm 1$
- c) Number of (+) real roots 2 or 0
- d) Number of (-) real roots 1
- e) Upper Bound of Roots 2
- f) Lower Bound of Roots -2
- g) Estimated value of the roots Between 0 & 1, 1 & 2

	4	1	-11	3
1)		4	5	-6
	4	5	-6	-3
2)	4	1	-11	3
upper		8	18	14
Bound	4	9	7	17
1)	-4	1	11	3
		-4	-3	8
	-4	-3	8	11
2)	-4	1	11	3
		-8	-14	-6
Lower	-4	-7	-3	-3
Bound				

$f(-2) = -3$ } root
 $f(-1) = 11$ -
 $f(0) = 3$ } root
 $f(1) = -3$ } -root
 $f(2) = 17$ } -root

$-1 < -2$

12. Find the number of possible positive real zeros and the number of possible negative real zeros and then determine the rational zeros.

$f(x) = x^4 + 2x^3 - 9x^2 - 2x + 8$ $f(-x) = x^4 - 2x^3 - 9x^2 + 2x + 8$

positive real roots = 2 or 0
 # negative real roots = 2 or 0
 Roots = 1, -1, -4, 2

Synthetic division is used to find roots

Rational functions – (vertical, horizontal, and slant asymptotes)

13. The function $f(x) = x^3 - 3x$ has a critical point when $x = 0$.

13. minimum

Identify the point as a maximum, a minimum, or a point of inflection, and state its coordinates.

at $x = -0.01$ $f(x) = 0.03$
 at $x = 0$ $f(x) = 0 \rightarrow$ below boundary values = minimum
 at $x = 0.01$ $f(x) = 0.03$

14. Determine the vertical and horizontal asymptotes for the graph of

14. $x = -3$
 $y = 1$

$y = \frac{x-4}{x+3}$

$x+3=0 \rightarrow x=-3$ (vertical asymptote)
 $y = \frac{\frac{x}{x} - \frac{4}{x}}{\frac{x}{x} + \frac{3}{x}} ; y = \frac{1-0}{1+0} ; y = 1$ (horizontal asymptote)

15. Find the slant asymptote and the vertical asymptote for

$$y = \frac{x^2 - x + 2}{x - 1}$$

$x - 1 = 0$; $x = 1$ vertical asymptote

15. $y = x$ slant

$$x - 1 \overline{) \begin{array}{r} x^2 - x + 2 \\ -(x^2 - x) \\ \hline 2 \end{array}}$$

$y = x$ is slant asymptote

2 ← remainder that does not matter

Transformations – (reflections, translations, and dilations)

16. Describe the transformation relating the graph of $y = (x - 1)^2 + 3$ to its parent function, $y = x^2$.

16. 1 right
3 upward

$$y = (x - 1)^2 + 3$$

↑ ↑

1 causes translation 1 to the right
3 causes translation 3 upward

17. Describe the transformation relating the graph of $y = (x - 1)^2$ to its parent function, $y = x^2$.

17. 1 right

↑
1 causes translation 1 to the right

Exponential Resolutions

18.
$$\frac{(2x^2y^{1/2})^{-6}(18x^3y^{-3})}{(3x^{-3}y^{-1})^3(27x^4y^2)} = \frac{1}{2592x^4}$$

$$\frac{(\frac{1}{64}x^{-12}y^{-2})(18x^3y^{-3})}{(27x^{-9}y^{-3})(27x^4y^2)}$$

19.
$$(4x^3y^6)^{-1/2}(9x^6y^2)^2 = \frac{81x^{21/2}}{2}$$

$$(4^{1/2}x^{-3/2}y^{-3})(81x^{12}y^4)$$

Solving growth/decay problems

20. The yield in millions of cubic feet of trees per acre is given by the equation $y = 6.7e^{-48.1/t}$ for a forest that is t years old. a) Find the year after 15 years. b) Find the yield after 50 years.

a) $y = 6.7e^{-\frac{48.1}{15}} = 0.27$ million ft^3 of trees

b) $y = 6.7e^{-\frac{48.1}{50}} = 2.56$ million ft^3 of trees

21. The population of Detroit was 1,785,000 in 1992. The city has experienced a decline in population that resulted in a 12% decrease for each year beyond this time. Assuming that this annual rate of decline continued for each year since 1992, what is the estimated population in 2010?

$$N = N_0(1+r)^t$$

$$N = 1785000(1 - 0.12)^{18}$$

$$N = 178,783$$

Logarithm Rules

$$22. \frac{1}{4}(\log_7 x + \log_7 8) = \frac{1}{4} \log_7 16$$

$$\frac{1}{4}(\log_7 8x) = \log_7 16^{\frac{1}{4}}$$

$$\log_7 (8x)^{\frac{1}{4}} = \log_7 2$$

$$(8x)^{\frac{1}{4}} = 2$$

$$8x = 2^4$$

$$8x = 16$$

$$x = \underline{2}$$

$$x = 2$$

$$23. \log_{12} x = \frac{1}{2} \log_{12} 36 + \frac{1}{3} \log_{12} 27$$

$$\log_{12} x = \log_{12} 36^{\frac{1}{2}} + \log_{12} 27^{\frac{1}{3}}$$

$$\log_{12} x = \log_{12} 6 + \log_{12} 3$$

$$x = \underline{18}$$

$$\log_{12} x = \log_{12} 18 \quad x = 18$$

$$24. 2 \log_3 9 - \frac{1}{2} \log_3 81 = x$$

$$\log_3 9^2 - \log_3 81^{\frac{1}{2}} = x$$

$$\log_3 81 - \log_3 9 = x$$

$$\log_3 \frac{81}{9} = x$$

$$\log_3 9 = x$$

$$x = \underline{2}$$

$$3^x = 9$$

$$3^x = 3^2$$

$$x = 2$$

Graphing of an Exponential Function

$$25. f(x) = 2^x - 1 \quad \{x: -3 \leq x \leq 3\}$$

x	f(x)
-3	-0.875
-2	-0.75
-1	-0.50
0	0
1	1
2	3
3	7

