Trig/Precalculus
Conic Sections
Mr. Roy

Name Key
Date ______
Period 3rd

For each given equation you are to: a) determine the type of conic section from the given equation, b) determine the standard form equation, c) determine and state all of the characteristics of this type of conic section, and d) graph the result and include all features of the conic section on the graph.

1. $6x^2 + 10y^2 + 48x + 20y = 14$

Type: Ellipse

Std. Equation: $\frac{(x+4)^2}{20} + \frac{(y+1)^2}{12} = 1$

Characteristics
Center $(-4,-1)$
major Vertices
Minor Vertices
Foci

$3x^2 + 24x + 5y^2 + 10y = 7$

$3(x^2 + 8x + 4^2) + 5(y^2 + 2y + 1^2) = 7 + 3(4^2) + 5(1^2)$

$3(x+4)^2 + 5(y+1)^2 = 7 + 48 + 5 = 60$

$\frac{3(x+4)^2}{60} + \frac{5(y+1)^2}{60} = \frac{60}{60}$

$\frac{(x+4)^2}{20} + \frac{(y+1)^2}{12} = 1$

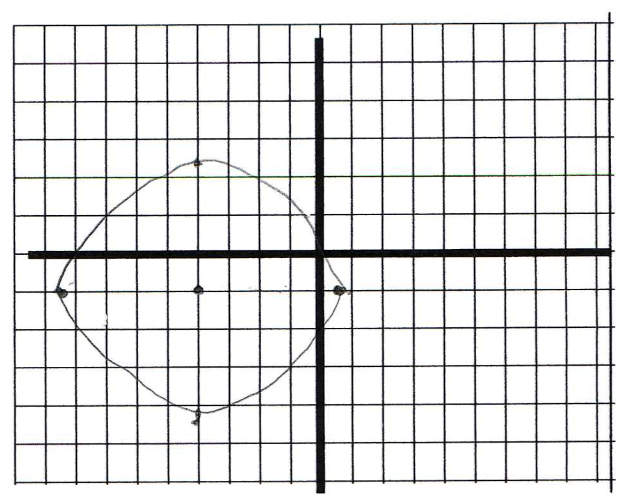
horizontal

$a^2 = 20;\ a = 4.5$
$b^2 = 12;\ b = 3.5$
$c^2 = 8;\ c = 2.8$
$h = -4$
$k = -1$

$(h \pm a, k)$
$(.5, -1)\ (-8.5, -1)$

$(h, k \pm b)$
$(-4, 2.5)\ (-4, -4.5)$

$(h \pm c, k)$



2. $x^2 + y^2 + 14x + 6y = -50$

$(x^2+14x+7^2)+(y^2+6y+3^2)=-50+7^2+3^2$

$(x+7)^2+(y+3)^2=8$

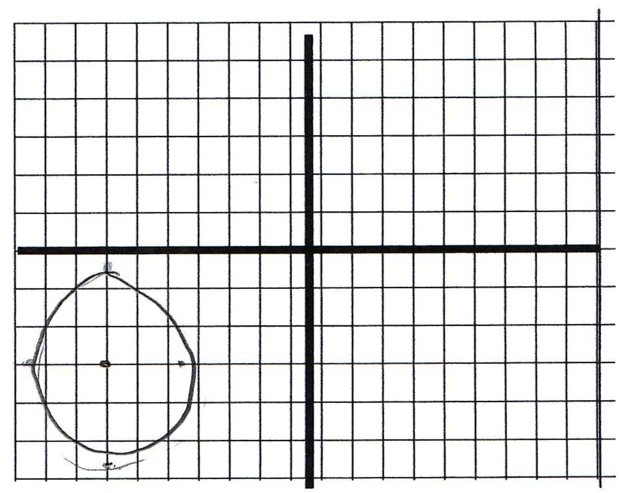
Type: Circle

Std. Equation: $(x+7)^2+(y+3)^2=8$

Characteristics

Center $(-7,-3)$

radius $\sqrt{8}=2.8$



3. $y^2 - 4x^2 - 2y - 16x = -1$

$y^2-2y+1^2-4(x^2+4x+2^2)=-1+1^2-16$

$\dfrac{(y-1)^2}{-16}-\dfrac{4(x+2)^2}{-16}=\dfrac{-16}{-16}$

$\dfrac{(x+2)^2}{4}-\dfrac{(y-1)^2}{16}=1$ horizontal

$h=-2$

$k=-1$

$a^2=4$; $a=2$

$b^2=16$; $b=4$

$c^2=20$; $c=4.5$

$(h\pm a, k)$

$(0,-1)(-4,-1)$

$(h\pm c, k)$

$(2.5,-1)(-6.5,-1)$

$(y-k)=\pm\frac{b}{a}(x-h)$

$(y+1)=\pm 2(x+2)$

Type: Hyperbola

Std. Equation: $\dfrac{(x+2)^2}{4}-\dfrac{(y-1)^2}{16}=1$

Characteristics

Center $(-2,-1)$

Vertices $(0,-1)\ (-4,-1)$

foci $(2.5,-1)\ (-6.5,-1)$

asymptote $(y+1)=\pm 2(x+2)$

4. $y = 3x^2 - 24x + 50$

Type Parabola

Std. Equation $(x-4)^2 = \frac{1}{3}(y-6)$

Characteristics

Vertex $(4, 6)$

focus $(4, 6\frac{1}{12}$

directrix $y = 5\frac{11}{12}$

$3x^2 - 24x = y - 50$

$3(x^2 - 8x + 4^2) = y - 50 + 3(4^2)$

$3(x-4)^2 = y - 2$

$(x-4)^2 = \frac{1}{3}(y-6)$ vertical

$h = 4$

$k = 6$

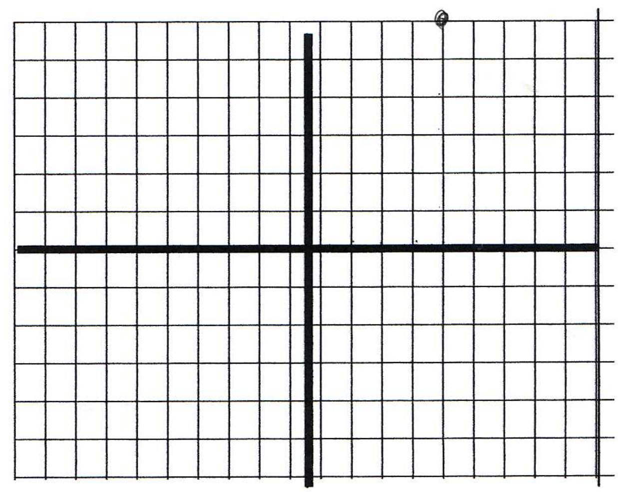
$4p = \frac{1}{3}$

$p = \frac{1}{12}$

(h, k) $(4, 6)$

$(h, k+p)$ $(4, 6\frac{1}{12})$

$y = 6 - \frac{1}{12} = 5\frac{11}{12}$



5. *Write the equation of the given circle in standard form. The center is (- 1, 2) and the radius is 3.*

$(x+1)^2 + (y-2)^2 = 3^2$

$(x+1)^2 + (y-2)^2 = 9$

6. *Determine the equation of an ellipse that has foci points of (1, - 2) and (5, -2) and the ellipse passes through the point at (3, - 1).* horizontal

$\frac{(x-3)^2}{5} + \frac{(y+2)^2}{1} = 1$

$k = -2$

$h = 3$

$c = 2$

$b = 1$

$a = \sqrt{5}$

$h + c = 5$

$h - c = 1$

$2h = 6$ $c = 2$

$h = 3$

$c^2 = a^2 - b^2 \rightarrow c^2 + b^2 = a^2$

$4 + 1 = a^2$

$5 = a^2$

7. *The vertex of the parabola is located at (1,-5) and the focus is located at (1,-3). Determine the equation of the parabola in standard form.*

$h = 1$

$k = -5$

$p = 2$

$(x-h)^2 = 4p(y-k)$

$(x-1)^2 = 8(y+5)$

8. *Find the equation of a hyperbola that has its foci at (7, 3) and (7,-1) with the transverse axis of length of 2.*

$$\frac{(y-1)^2}{1} - \frac{(x-7)^2}{3} = 1$$

$2a = 2$; $a = 1$
$h = 7$, $k = 1$; $c = 2$
$k + c = 3$; $b = \sqrt{3}$
$k - c = -1$
$2k = 2$
$k = 1$ $c = 2$
vertical

$c^2 = a^2 + b^2$
$b^2 = c^2 - a^2$
$b^2 = 4 - 1 = 3$

9. *Find the equation of a parabola that has a vertex of (1, 4) and a directrix of $y = 1$.*

$$(x-1)^2 = 12(y-4)$$

$h = 1$; $1 = k - p$
$k = 4$; $1 = 4 - p$
vertical $p = 3$

Horizontal	***Parabola***	***Vertical***
$(y-k)^2 = 4p(x-h)$	***Standard Formula***	$(x-h)^2 = 4p(y-k)$
(h,k)	***Vertex***	(h,k)
$(h+p, k)$	***Focus***	$(h, k+p)$
$x = h-p$	***Directrix***	$y = k-p$
$y = k$	***Axis of Symmetry***	$x = h$

Horizontal	***Hyperbola***	***Vertical***
$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$	***Standard Equation***	$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$
$(h+a, k)\ (h-a, k)$	***Vertices***	$(h, k+a)\ (h, k-a)$
$(h+c, k)\ (h-c, k)$	***Foci***	$(h, k+c)\ (h, k-c)$
$(y-k) = \pm\frac{b}{a}(x-h)$	***Asymptote line***	$(y-k) = \pm\frac{a}{b}(x-h)$

	Circle
$(x-h)^2 + (y-k)^2 = r^2$	***Standard Equation***
(h,k)	***Center***
r	***radius***

Horizontal	***Ellipse***	***Vertical***
$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$	***Standard Equation***	$\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$
(h,k)	***Center***	
$(h+a, k)\ (h-a, k)$	***Major Vertices***	$(h, k+a)\ (h, k-a)$
$(h, k+b)\ (h, k-b)$	***Minor Vertices***	$(h+b, k)(h-b, k)$
$(h+c, k)\ (h-c, k)$	***Foci***	$(h, k+c)\ (h, k-c)$