Trigonometry/Pre-calculus
Hyperbolas, Ellipses, & Circles
Mr. Roy

Name Key
Date ______
Period ______

1. $9x^2 - 8y^2 + 36x - 16y - \cancel{100}\ 116 = 0$ *Put this equation in proper form and determine all characteristics. Graph the equation.*

$9(x^2+4x+2^2) - 8(y^2+2y+1^2) = 116 + 36 - 8$

$\frac{9(x+2)^2}{144} - \frac{8(y+1)^2}{144} = \frac{144}{144}$ horizontal

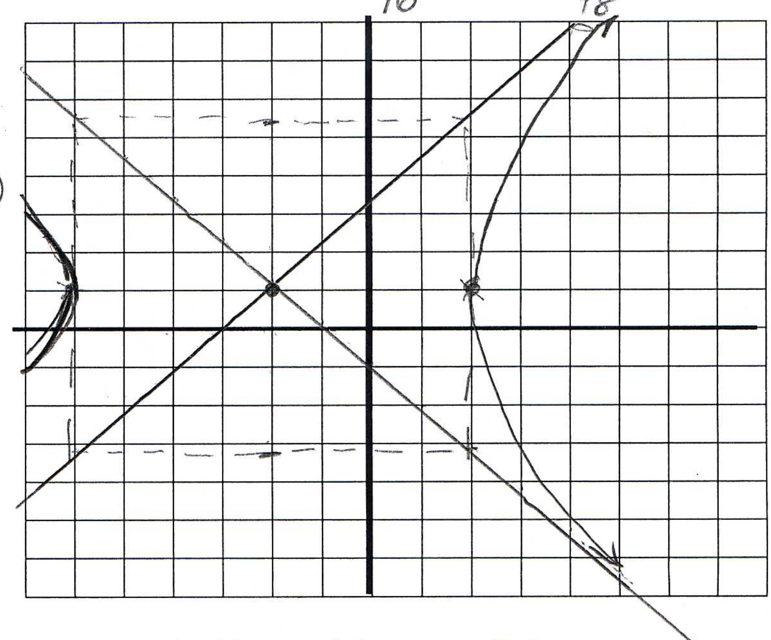
$\frac{(x+2)^2}{16} - \frac{(y+1)^2}{18} = 1$

$h=-2$
$k=-1$
$a^2=16$; $a=4$
$b^2=18$; $b=4.2$ $(3\sqrt{2})$
$c^2=34$; $c=5.8$ $(\sqrt{34})$

$(h\pm c, k)$ $(-2\pm\sqrt{34}, -1)$
$(h\pm a, k)$ $(-2\pm 4, -1)$
$(2,1)$ $(-6,-1)$

$y-k = \pm\frac{b}{a}(x-h)$
$y+1 = \frac{3}{4}\sqrt{2}(x+2)$

Characteristics: Center $(-2, 1)$ Foci $(-2+\sqrt{34}, -1)$ $(-2-\sqrt{34}, -1)$
Vertices $(2, 1)$ $(-6, 1)$ Asymptotes $y+1 = \pm\frac{3}{4}\sqrt{2}(x+2)$
Standard equation $\frac{(x+2)^2}{16} - \frac{(y+1)^2}{18} = 1$



2. $x^2 - 8x + y^2 - 6y - 24 = 0$ *Put this equation in standard form and determine all characteristics.*

$(x^2-8x+4^2)+(y^2-6y+3^2) = 24+16+9$

$(x-4)^2+(y-3)^2 = 49$

Characteristics Center $(4, 3)$ *radius* = 7

3. *A vertex of the hyperbola is located at (-5, 1) and a focus is located at (-8, 1). The center of the hyperbola is located at (-1, 1). Determine the equation of the hyperbola in standard form.*

$h=-1$
$k=1$
$a=4$
$c=7$

$h-a=-5$
$-1-a=-5$
$a=4$
horizontal

$h-c=-8$
$-1-c=-8$
$c=7$

$b=\sqrt{33}$

$a^2=16$
$c^2=49$
$b^2=33$

$\frac{(x+1)^2}{16} - \frac{(y-1)^2}{33} = 1$

4. *Find the equation of a hyperbola that has its foci at (5, 2) and (-5, 2) with the transverse axis of length of 6.*

$k = 2 \quad h = 0 \quad c = 5 \quad a = 3$

$h + c = 5$
$h - c = -5$
$2h = 0$

$b = \sqrt{25 - 9} = \sqrt{16}$
$b = 4$
horizontal

$$\frac{(x-0)^2}{9} - \frac{(y-2)^2}{16} = 1$$

5. *Given the equation, find the coordinates of the center, foci, and vertices.*

$$\frac{(x-7)^2}{36} + \frac{(y-5)^2}{24} = 1$$

Center $(7, 5)$
Foci $(7+\sqrt{12}, 5)$ $(7-\sqrt{12}, 5)$
Major Vertices $(13, 5)$ $(1, 5)$
Minor Vertices $(7, 5+\sqrt{24})$ $(7, 5-\sqrt{24})$

$h = 7$
$k = 5$
$a^2 = 36$; $a = 6$
$b^2 = 24$; $b = \sqrt{24}$
$c^2 = 12$
$c = \sqrt{12}$

Horizontal	***Hyperbola***	***Vertical***
$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$	***Standard Equation***	$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$
$(h + a, k)$ $(h - a, k)$	***Vertices***	$(h, k + a)$ $(h, k - a)$
$(h + c, k)$ $(h - c, k)$	***Foci***	$(h, k + c)$ $(h, k - c)$
$(y - k) = \pm\frac{b}{a}(x - h)$	***Asymptote line***	$(y - k) = \pm\frac{a}{b}(x - h)$

	Circle
$(x-h)^2 + (y-k)^2 = r^2$	***Standard Equation***
(h,k)	***Center***
r	***radius***

Horizontal	***Ellipse***	***Vertical***
$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$	***Standard Equation***	$\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$
(h,k)	***Center***	
$(h + a, k)$ $(h - a, k)$	***Major Vertices***	$(h, k + a)$ $(h, k - a)$
$(h, k + b)$ $(h, k - b)$	***Minor Vertices***	$(h+ b, k)(h - b, k)$
$(h + c, k)$ $(h - c, k)$	***Foci***	$(h, k + c)$ $(h, k - c)$