Section 4.7 Radical Equations and Inequalities

solving radical equations requires that first the radical be isolated.

Afterwards, raise both sides by an exponent that will negate the radical and solve for the value of the variables.

Some of the resulting values lead to extraneous solutions which requires that all values be checked for correctness.

Ex. $4 = \sqrt[3]{x+2} + 8$

$-8 \quad -8$

$-4 = \sqrt[3]{x+2}$

$(-4)^3 = (\sqrt[3]{x+2})^3$

$-64 = x+2$

$-2 \quad -2$

$-66 = x$

check:

$4 = \sqrt[3]{-66+2} + 8$

$4 = \sqrt[3]{-64} + 8$

Ex. $\sqrt{x+16} = \sqrt{x} + 4$

$(\sqrt{x+16})^2 = (\sqrt{x}+4)^2$

$x+16 = x + 8\sqrt{x} + 16$

$x+16-x-16 = 8\sqrt{x}$

$0 = 8\sqrt{x}$

$0 = x$

check

$\sqrt{0+16} = \sqrt{0} + 4$

$\sqrt{16} = 0 + 4$

$4 = 4$ ✓

When radical expressions are placed into an inequality, it is necessary to consider minimum values under the radical to prevent creating imaginary values.

Ex. $\sqrt{4x+5} \leq 10$

$\left(\sqrt{4x+5}\right)^2 \leq (10)^2$

$4x+5 \leq 100$

$4x \leq 95$

$x \leq \frac{95}{4}$

Looking for the minimum

$4x+5 \geq 0$

$4x \geq -5$

$x \geq \frac{-5}{4}$

Solution: $\frac{-5}{4} \leq x \leq \frac{95}{4}$

Ex: $\sqrt{2x-7} \geq 5$

$2x-7 \geq 25$

$2x \geq 32$

$x \geq 16$

with the root $x \geq 16$, 16 is the minimum. we do not need to find the minimum of the radical.

Ex. $\sqrt{m+2} \leq \sqrt{3m+4}$

$m+2 \leq 3m+4$

$2 \leq 2m+4$

$-2 \leq 2m$

$-1 \leq m$

$m \geq -1$

with $m \geq -1$

-1 is the minimum, no other values need to be found.