

Section 4-3 The Remainder and Factor Theorems

Consider a function $f(x) = x^3 + 3x^2 - 5x + 7$ and a divisor of $x - 1$

$$\begin{array}{r} x^2 + 4x - 1 \\ x-1 \overline{) x^3 + 3x^2 - 5x + 7} \\ \underline{-(x^3 - x^2)} \\ 4x^2 - 5x + 7 \\ \underline{-(4x^2 - 4x)} \\ -x + 7 \\ \underline{-(-x + 1)} \\ 6 \end{array}$$

Notice that the remainder from the division is 6. If we consider the same function evaluated at $x = 1$, $f(1)$, we will get a value of 6.

$$f(1) = 1^3 + 3(1)^2 - 5(1) + 7 = 1 + 3 - 5 + 7 = 6$$

This outcome leads to the Remainder Theorem.

If a polynomial $P(x)$ is divided by $x - r$, the remainder is a constant $P(r)$, and $P(x) = (x - r) \cdot Q(x) + P(r)$ where $Q(x)$ is a polynomial with degree one less than the degree of $P(x)$.

r is the value to be evaluated in $P(x)$.

$Q(x)$ the resulting quotient

$P(r)$ the remainder from the division.

