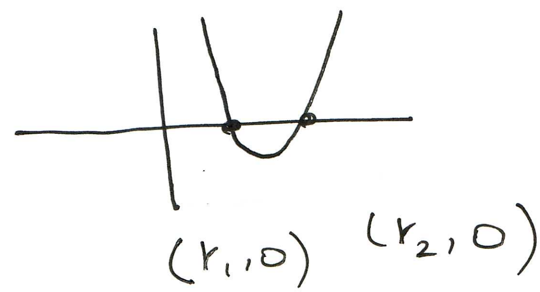
4.2 Quadratic Equations

Quadratic equations have a degree of 2 and as such have 2 roots. The solution of quadratic equations can be determined in four different ways.

First - By graphing



Second - By factoring

$$y = x^2 + 2x - 3$$

$$(x+3)(x-1) = 0 \qquad x = -3, 1$$

Third - By completing the square

change the equation into a statement that is a perfect square and then solve for the roots.

$$x^2 + 4x - 6 = 0 \quad \rightarrow \quad x^2 + 4x = 6$$

$$x^2 + 4x + 2^2 = 6 + 2^2 \quad \rightarrow \quad (x+2)^2 = 10$$

$$x + 2 = \pm\sqrt{10} \quad \rightarrow \quad x = -2 \pm \sqrt{10}$$

Fourth - By use of the Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad x^2 + 4x - 6 = 0; \quad a = 1, \; b = 4, \; c = -6$$

$$x = \frac{-4 \pm \sqrt{4^2 - 4(1)(-6)}}{2(1)} = \frac{-4 \pm \sqrt{16 + 24}}{2} = \frac{-4 \pm \sqrt{40}}{2}$$

$$x = \frac{-4 \pm 2\sqrt{10}}{2}; \quad \boxed{x = -2 \pm \sqrt{10}}$$

Completing the Square Technique

① make sure that the lead coefficient is + 1 and that the constant is placed on the other side of the equal sign.

② using the coefficient of the middle (linear term), take $\frac{1}{2}$ of that value, square it and add this amount to both sides.

③ Factor as a perfect square and combine terms on the other side.

④ Square root both sides and solve for the two values of x.

Ex. 1 $x^2 - 8x + 10 = 0$

① $x^2 - 8x = -10$

② $x^2 - 8x + 4^2 = -10 + 4^2$

③ $(x-4)^2 = 6$

④ $x - 4 = \pm\sqrt{6} \rightarrow \boxed{x = 4 \pm \sqrt{6}}$

Ex. 2. $3x^2 + 6x - 27 = 0$

① $x^2 + 2x = 9$ (Divide through by 3)

② $x^2 + 2x + 1^2 = 9 + 1^2$

③ $(x+1)^2 = 10$

④ $x + 1 = \pm\sqrt{10} \rightarrow x = -1 \pm \sqrt{10}$

Quadratic Formula

using the coefficients of a quadratic polynomial to determine the zeroes of the function. This technique is especially helpful when the lead coefficient is not a 1.

Ex. $4x^2 - 5x + 7 = 0$

$a = 4$
$b = -5$
$c = 7$

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(4)(7)}}{2(4)} = \frac{5 \pm \sqrt{25 - 112}}{8} = \frac{5 \pm \sqrt{-87}}{8}$$

$$x = \frac{5 \pm i\sqrt{87}}{8}$$

Ex #2 $2x^2 + 11a - 21 = 0$

$a = 2$
$b = 11$
$c = -21$

$$x = \frac{-11 \pm \sqrt{11^2 - 4(2)(-21)}}{2(2)}$$

$$x = \frac{-11 \pm \sqrt{121 + 168}}{4} = \frac{-11 \pm \sqrt{289}}{4} = \frac{-11 \pm 17}{4};$$

$$x = \frac{-11 + 17}{4} = \frac{3}{2}$$

$$x = \frac{-11 - 17}{4} = -7$$

$x = \frac{3}{2}, -7$

In the quadratic formula, the radicand, b^2-4ac is called the <u>discriminant</u> of the equation. The discriminant value can be used to determine the nature of the roots.

If $b^2-4ac > 0$ then there are 2 real roots
If $b^2-4ac = 0$ then there is 1 real repeated root.
If $b^2-4ac < 0$ then there are 2 imaginary roots.

Ex. $4x^2-5x+7=0$ $\quad (-5)^2-4(4)(7)=25-112=-87$
Discriminant <0, so there are 2 imaginary roots.

Ex. $2x^2+11a-21=0$ $\quad (11)^2-4(2)(-21)=121+168=289$
Discriminant >0, so there are 2 real roots

Ex $36d^2-84d+49=0$ $\quad (-84)^2-4(36)(49)=7056-7056=0$
Discriminant $=0$, so there is 1 repeating real root.

Ex. page 219-220

#14. $x^2 - 10x + 21 = 0$

a) Factoring $(x-7)(x-3) = 0 \rightarrow x = 7, 3$

b) Complete the square

$$x^2 - 10x = -21$$

$$x^2 - 10x + 5^2 = -21 + 5^2$$

$$(x-5)^2 = 4$$

$$x - 5 = \pm\sqrt{4}$$

$$x = 5 \pm 2 \rightarrow x = 7, 3$$

c) Quadratic Formula $x^2 - 10x + 21 = 0$

$$x = \frac{-(-10) \pm \sqrt{(-10)^2 - 4(1)(21)}}{2}$$

$$x = \frac{10 \pm \sqrt{100 - 84}}{2} = \frac{10 \pm \sqrt{16}}{2} = \frac{10 \pm 4}{2}$$

$$x = \frac{14}{2}, \frac{6}{2} \rightarrow x = 7, 3$$

#23 $4x^2 - 2x + 9 = 0$ discriminant

2 imaginary roots $(-2)^2 - 4(4)(9) = 4 - 144 = -140$

$$x = \frac{2 \pm \sqrt{-140}}{8} ; \quad x = \frac{2 \pm 2i\sqrt{35}}{8} ;$$

$$x = \frac{1 \pm i\sqrt{35}}{4}$$