Section 4.1 Polynomial Functions

Polynomial functions with a single variable and three or more terms will be the topic of our study.

If we define a polynomial in one value, x, the expression is defined as: $P(x) = a_n x^n + a_{n-1} x^{n-1} + \dots a_2 x^2 + a_1 x + a_0$ where $a_n, a_{n-1}, a_2, a_1,$ and a_0 represent complex numbers, $a_n \neq 0$, and n represents a nonnegative integer. These values are coefficients and a_n is the <u>leading coefficient</u>. The largest exponent on the variable represents the <u>degree</u> of the polynomial.

Ex. $5x^4 - 2x^3 + 6x - 4$ is a polynomial with a lead coefficient of 5 and a degree of 4.

For each polynomial, there exists values for the variable, x, that cause the polynomial to equal zero. These values of x are called the zeros of the function.

Zero values of functions, roots, are complex numbers. Complex number can be real and/or imaginary. Imaginary numbers are based on $\sqrt{-1} = i$. We see that $\sqrt{-1} = i$, $i^2 = -1$, $i^3 = -\sqrt{i}$, $i^4 = 1$.

Imaginary numbers that exist as roots of a polynomial always come in pairs that are called <u>conjugate pairs</u>. $a + bi$; $a - bi$ a and b are real numbers and i is an imaginary number.

Ex. $6 + 2i$; $6 - 2i$

Fundamental Theorem of Algebra -

Every polynomial equation with degree greater than 0 has at least one root in the set of complex number.

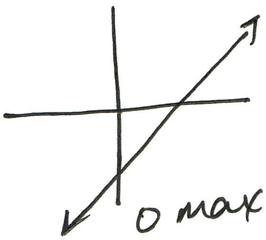
For $P(x) = a_n x^n + a_{n-1} x^{n-1} + \dots a_2 x^2 + a_1 x + a_0$ where $n > 0$, there will be n number of roots.

Also, Each polynomial $P(x)$ where $n > 0$ that can be written in the form $P(x) = k(x - r_1)(x - r_2)\dots(x - r_n)$ where $k \neq 0$, then $r_1, r_2, \dots r_n$ will be complex roots of $P(x)$.

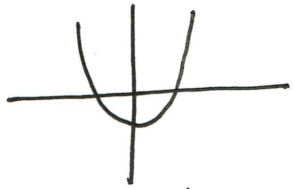
Ex. $P(x) = 2x^4 - 3x^3 + 4x^2 + 5x - 6$; the degree of $P(x)$ is 4. The highest exponent of x is 4. There will be 4 complex roots of $P(x)$.

The graphs of functions reveals the location and nature of its roots. where the graph intersects the x-axis is a real root. If the graph shows maximum or minimums, these also describe the roots. We see that the number of maximums/minimums + 1 is equal to the degree of the polynomial and the the number of real/imaginary roots.

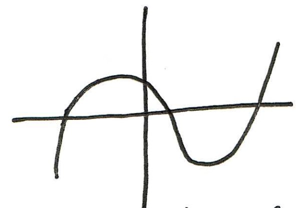
Example Graphs.



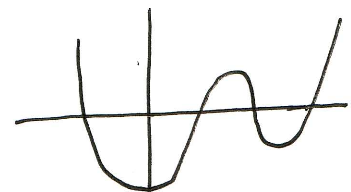
0 max/min
degree = 1



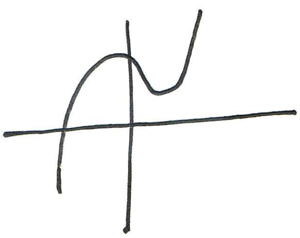
1 min
degree = 2



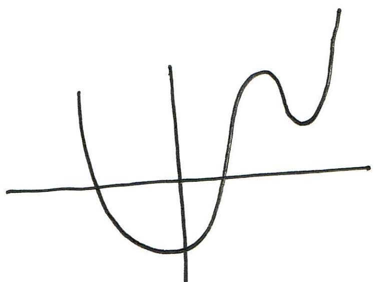
1 max / 1 min
degree = 3



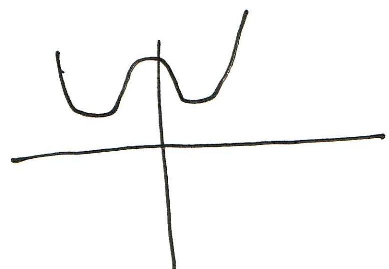
1 max / 2 min
degree = 4



degree = 3
1 real root
2 imaginary roots



degree = 4
2 real roots
2 imaginary roots



degree = 4
0 real roots
2 pairs imaginary roots

sample problems from pages 210-211

state the degree and leading coefficient of each polynomial.

#17. $9a^2 + 5a^3 - 10 \rightarrow 5a^3 + 9a^2 - 10$ degree = $\underline{3}$ lead coefficient = $\underline{5}$

Determine whether each number is a root of

$a^4 - 13a^2 + 12a = 0$ (-3)

$(-3)^4 - 13(-3)^2 + 12(-3) = 0 \rightarrow 81 - 117 - 36 = -72$

(-3) is not a root of the polynomial

Write out the P(x) whose roots are given.

35. $r_1 = -3, \quad r_2 = -2i, \quad r_3 = 2i$

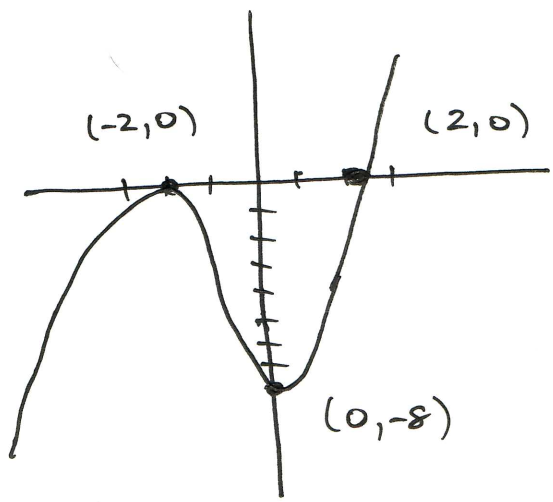
$$P(x) = (x - -3)(x - -2i)(x - 2i) = (x+3)(x+2i)(x-2i)$$
$$= (x+3)(x^2 - 4i^2) = (x+3)(x^2 - 4(-1)) = (x+3)(x^2+4)$$
$$= x^3 + 4x + 3x^2 + 12 = \boxed{x^3 + 3x^2 + 4x + 12}$$

42. state the # of complex roots, find the roots, graph the function.

$t^3 + 2t^2 - 4t - 8 = 0$

roots = 3

roots = -2, -2, 2



44. $6c^3 - 3c^2 - 45c = 0$

roots = 3

roots: -2.5, 0, 3