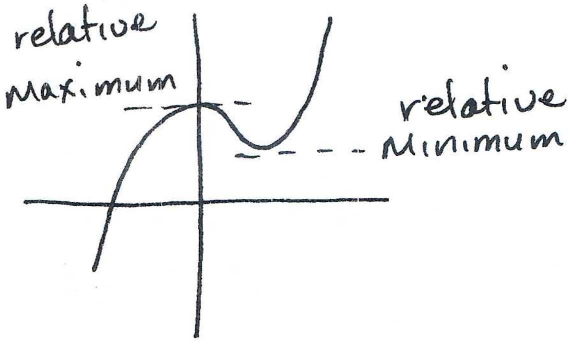
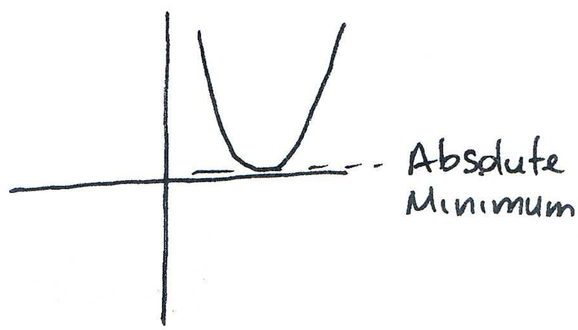
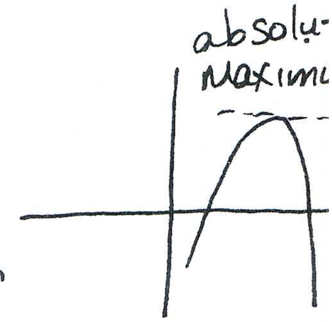
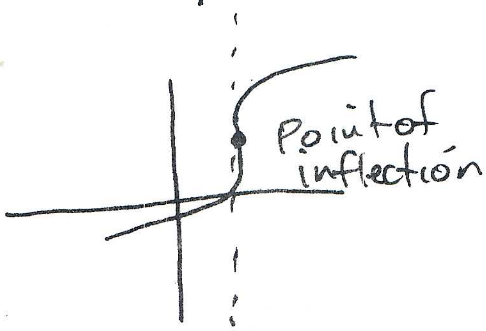
Section 3.6 Critical Points and Extrema

Critical points of a function are those values of x that cause the function to make changes in its direction. These changes can be seen as a peak, a trough, or even a value where the slope of the function becomes undefined. The peaks are called Maximums. The troughs are called minimum and the points with undefined slopes are points of inflection.

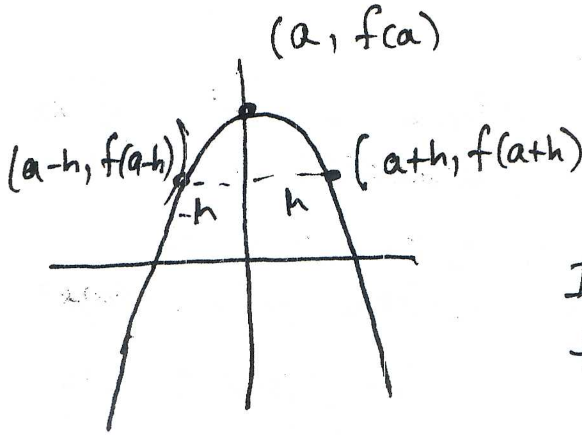






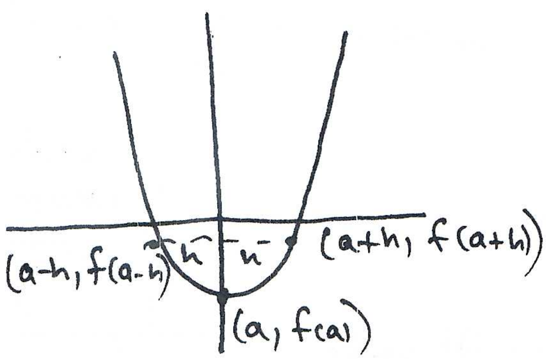


When visualizing the graph of a function, it is easy see these points. In the absence of the graph, a process of equation analysis is needed.

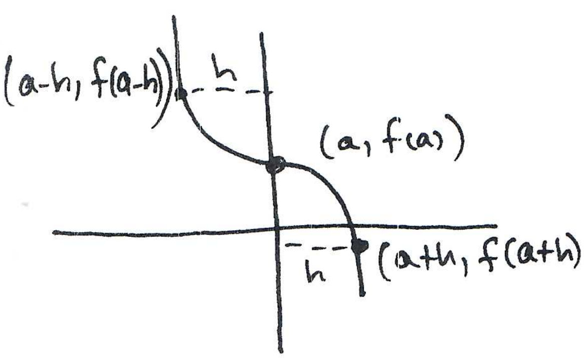


Consider a value that is a sm
increment (h) away from the
point (a, f(a)) on either side o

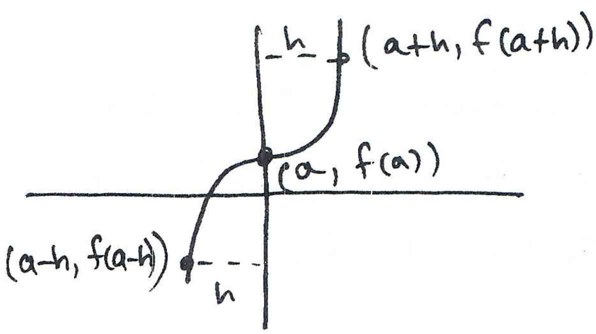
If $f(a-h) < f(a)$ and $f(a+h) < f(a$
then $(a, f(a))$ is a maximum.



If $f(a-h) > f(a)$ and $f(a+h) > f(a)$
then $(a, f(a))$ is a minimum.



If $f(a-h) > f(a)$ and $f(a+h) < f(a)$
then $(a, f(a))$ is a point of inflectio



If $f(a-h) < f(a)$ and $f(a+h) > f(a$
then $(a, f(a))$ is a point of inflect

Application Example

Given: $f(x) = 2x^2 + 10x - 7$ and $x = -2.5$ is a critical

Let $h = 0.1$ $a+h = -2.4$ $a-h = -2.6$

① Solve for $f(-2.5) = -19.5$

② Solve for $f(-2.4) = -19.48$

③ Solve for $f(-2.6) = -19.48$

④ Note that $f(-2.5)$ is below the other two values. Therefor $(-2.5, -19.5)$ must be a minimum.

Application Example #2

Given: $f(x) = x^4 - 2x^2 + 7$ and $x = 0$ is a critical point.

Let $h = 0.1$ $a+h = 0.1$ $a-h = -0.1$

① Solve for $f(0) = 7$

② Solve for $f(0.1) = 6.98$

③ Solve for $f(-0.1) = 6.98$

④ Note that $f(0)$ is above the other two values. Therefore $(0, 7)$ must be a maximum.