3-5 Continuity of Functions

A function is continuous at $x = c$ if it satisfies three conditions:

1. The function is defined at c; $f(c)$ exists
2. The function approaches the same y-value on left and right sides of $x = c$
3. The y-value that the function approaches from each side is $f(c)$.

Discontinuity occurs when the graph has a break at a finite value of x for $f(x)$ or becomes undefined at a finite value of x for $f(x)$

Ex. (a) $f(x) = \frac{1}{x^2}$ at $x = 0$ $f(x)$ is undefined

(b) $f(x) = x^2 + 1$ if $x < 0$
$= x$ if $x > 0$
at $x = 0$ $f(x)$ is not included for either function

(c) $f(x) = \frac{x^2 - 1}{x + 1}$ at $x = -1$ $f(x)$ is undefined but is continuous everywhere else

a) Infinite discontinuity as x approaches $0, +\infty, -\infty$ the function infinitely approaches $y = +\infty, -\infty, 0$. but never reaches finite values.

b) Jump discontinuity - The graph stops a point and begins again at a different y-value for the same x-value.

Point discontinuity - when a domain value for a function is undefined, but there is only one $f(x)$ for each x.

$f(x) = \frac{x}{x+1}$ at $x=-1$ function is undefined and is defined everywhere else with a one x to each $f(x)$ relationship.

Everywhere discontinuous - Functions cannot be graphed in the real number system.

$$f(x) = \begin{cases} 1 & \text{if } x \text{ is rational} \\ -1 & \text{if } x \text{ is irrational} \end{cases}$$

- rational: $-5, -2, 0, 1, 1\frac{1}{2}, 5$
- irrational: $\sqrt{2}, \pi, \sqrt{7}, \sqrt[3]{6}$

Review example 1

<u>Continuity on an Interval</u> - A function $f(x)$ is continuous on an interval if and only if it is continuous at each number x in the interval

Ex. Piecewise function - fails test

$$g(x) = \begin{cases} f(x) = 3x - 2 & \text{if } x > 2 \\ f(x) = 2 - x & \text{if } x \le 2 \end{cases}$$

at $x > 2$ continuous
at $x < 2$ continuous
at $x = 2$ Jump discontinuity because for $f(x) = 3x-2$ $x > 0$ $f(x)$ approaches 4 and for $f(x) = 2-x$ $x \le 2$ $f(x) = 0$ The jump between 4 & 0 is jump discontinuity.

End Behavior – Describes what the y-values do as $|x|$ approaches $\pm\infty$.

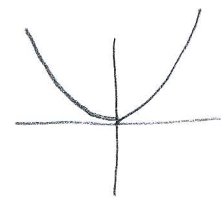
End behavior of polynomial functions.

$$P(x) = a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \dots + a_2 x^2 + a_1 x + a_0,\ n:$$

If a_n is (+) and n is even

$x \to +\infty \quad f(x) \to +\infty$

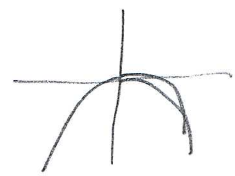
$x \to -\infty \quad f(x) \to -\infty$



If a_n is (−) and n is even

$x \to +\infty \quad f(x) \to -\infty$

$x \to -\infty \quad f(x) \to -\infty$

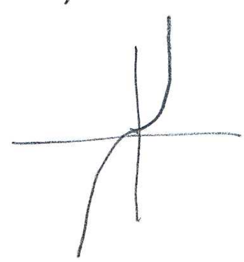


reflection

If a_n is (+) and n is odd

$x \to +\infty \quad f(x) \to +\infty$

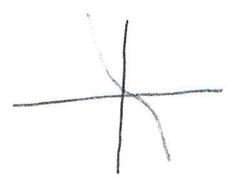
$x \to -\infty \quad f(x) \to -\infty$



If a_n is (−) and n is odd

$x \to \infty \quad f(x) \to -\infty$

$x \to -\infty \quad f(x) \to \infty$



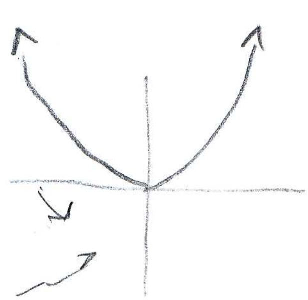
reflection

Monotonicity – An expression of whether a function is increasing or decreasing over an interval;

Ex. $f(x) = x^2$

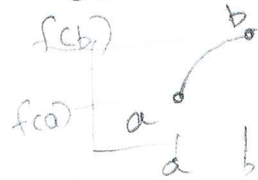
for $x < 0$ decreasing

for $x > 0$ increasing

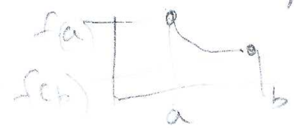


monotinicity

A function f is increasing on I if and only if for every a and b contained on I, $f(a) < f(b)$ whenever $a < b$.



A function f is decreasing on I if and only if for every a and b contained on I, $f(a) > f(b)$ when every $a < b$.



A function remains constant on I if and only if for every a and b contained on I, $f(a) = f(b)$ when every $a < b$.

Page 166 #13. $y = \dfrac{x+1}{x-2}$; $x = 2$ no because it is undefined at $x = 2$

#17. $f(x) = 2x+1$ if $x \geq 1$ yes function is defined at 1
$4 - x^2$ if $x < 1$ $f(x)$ approaches 3 as x approaches 1 from both sides and $f(1) = 3$

#23. $g(x) = |(x-3)^2 - 1|$ $g(x) \to y \to \infty$ as $x \to \infty$
$y \to \infty$ as $x \to -\infty$