Chapter 3.4 Inverse Functions and Relations

Two relations are inverses if $(a,b) \in f(x)$ and $(b,a) \in f^{-1}(x)$.

This means that the ordered pairs (a,b) for $f(x)$ will be reversed to (b,a) for $f^{-1}(x)$.

Also, $f^{-1}(x)$ can be determined algebraically by reversing x & y in $f(x)$ and then solving for y.

Inverse functions will be symmetric to $y=x$.

To check if an inverse relation is a function use the horizontal line test. If each range value has only one domain value, the inverse relation is a function.

This can be tested and confirmed algebraically by seeing if $(f \circ f^{-1})(x) = x$. If this equation works, then $f(x)$ and $f^{-1}(x)$ are functions.

$(f^{-1} \circ f)(x) = x$ also

Inverse Relations

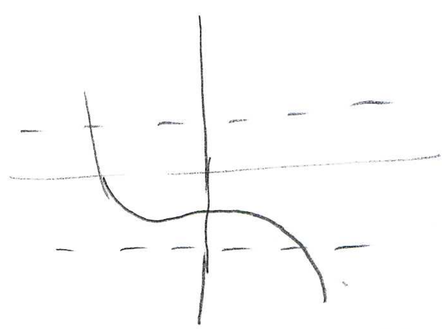
Two relations are inverse relations iff one relation contains (a,b) and its inverse contains the element (b,a)

For $f(x)$ with element (a,b) then $f^{-1}(x)$ has to have (b,a) as an element.

If $f(x)$ is a function then $f^{-1}(x)$ is a function if $f^{-1}(x)$ passes the vertical line test.

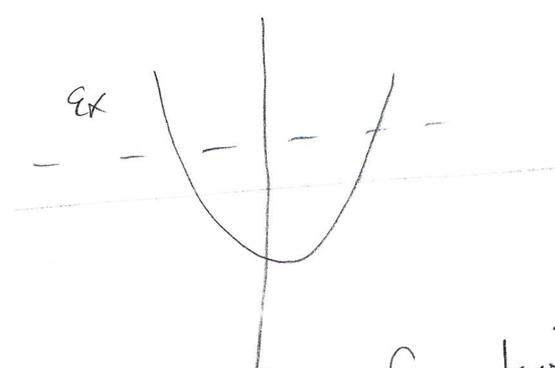
If $f(x)$ is a relation then $f^{-1}(x)$ is a function if $f^{-1}(x)$ passes the horizontal line test.

Ex.



$g(x)$: $g(x)$ passes horizontal line test, so $g^{-1}(x)$ will be a function

Ex



$g(x)$ is a function but $g^{-1}(x)$ is not.

The inverse of $f(x)$, $f^{-1}(x)$ is the reflection of $f(x)$ over the line $y=x$

Inverse Functions

Two functions $f(x)$ and $f^{-1}(x)$ are inverse functio
if and only if $(f \circ f^{-1})(x) = (f^{-1} \circ f)(x) = x$

Both functions pass the vertical line test and their composites in either direction equal x.

Ex. $f(x) = 4x - 9$

$y = 4x - 9$ $\quad x = 4y - 9$

$\frac{x+9}{4} = y$

$f(x) = 4x - 9 \quad f^{-1}(x) = \frac{x+9}{4}$

$(f \circ f^{-1})(x) =$

$4\left(\frac{x+9}{4}\right) - 9 = x + 9 - 9 = x$

$(f^{-1} \circ f)(x) = \frac{(4x-9)+9}{4} = \frac{4x}{4} = x$

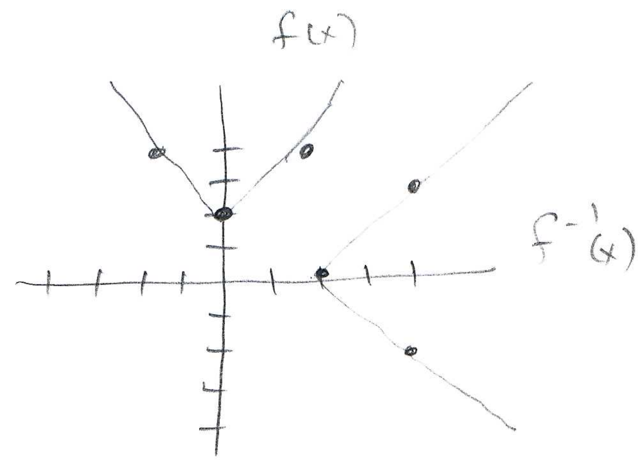
$f(x)$ and $f^{-1}(x)$ are inverse functions

Page 156

15. $f(x) = |x| + 2$

x	$f(x)$
-2	4
0	2
2	4

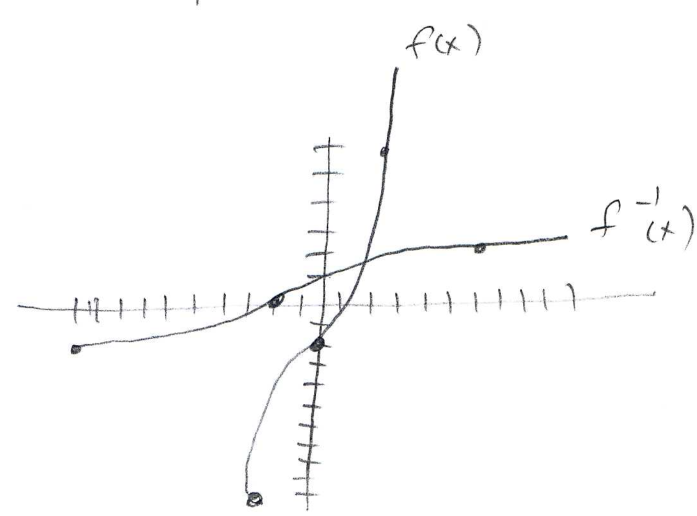
x	$f^{-1}(x)$
4	-2
2	0
4	2



17. $f(x) = x^3 - 2$

x	$f(x)$
-2	-10
0	-2
2	6

x	$f^{-1}(x)$
-10	-2
-2	0
6	2



21. $f(x) = x^2 + 2x + 4$

$y = x^2 + 2x + 4$

$f^{-1}(x) \quad x = y^2 + 2y + 4$

$x - 4 + 1^2 = y^2 + 2y + 1^2$

$f^{-1}(x) = -1 \pm \sqrt{x-3}$

$x - 3 = (y+1)^2$

$\pm\sqrt{x-3} = y + 1$

$-1 \pm \sqrt{x-3} = y$

function test for $f(x)$ & $f^{-1}(x)$

$(f \circ f^{-1})(x) = (-1+\sqrt{x-3})^2 + 2(-1+\sqrt{x-3}) + 4$

$= 1 - 2\sqrt{x-3} + x - 3 - 2 + 2\sqrt{x-3} + 4$

$= x$

$(f^{-1} \circ f)(x) = -1 + \sqrt{(x^2+2x+4)-3} = -1 + \sqrt{x^2+2x+1}$

$= -1 + \sqrt{(x+1)^2} = -1 + x + 1 = x$