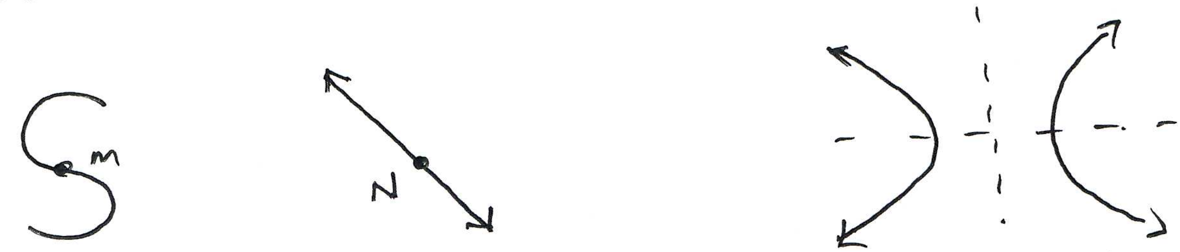
3-1 Symmetry & Coordinate Graphs

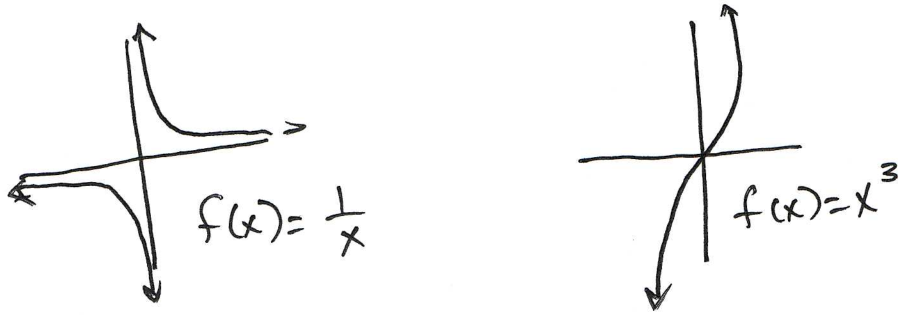
Point Symmetry (midpt of a segment) - Two distinct points P & P' are symmetric to point M iff M is the midpoint of PP'.

when point symmetry is extended to a set of points like a function, then each point in the set must have an image point also in the set.

A symmetric figure to a given point can be rotated 180° about that point and appear unchanged.

ex.





Symmetry with Respect to the Origin - The graph of relation S is symmetric with respect to the origin iff $(a, b) \in S$ implies that $(-a, -b) \in S$. A function has a graph that is symmetric with respect to the origin iff $f(-x) = -f(x)$ for all x in the domain of f.

Ex. $f(x) = x^2 - 2x - 1$ then $f(-x) = x^2 + 2x - 1$ and

$-f(x) = -x^2 + 2x + 1$

not symmetric with respect to the origin

Ex. If. $f(x) = -3x^3 + 5x$

$f(-x) = -3(-x)^3 + 5(-x) = +3x^3 - 5x$

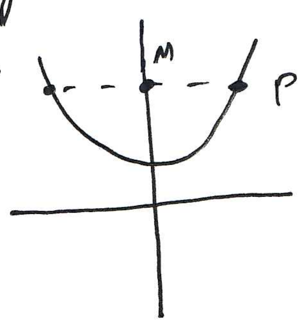
$-f(x) = -(-3x^2 + 5x) = 3x^2 - 5x$

Since $f(-x)$ and $-f(x)$ are the same, the function is symmetric to a point. (the origin)

Line Symmetry

Two points P and P' are symmetric with respect to a line l iff l is the perpendicular bisector of $\overline{PP'}$. A point P is symmetric to itself with respect to line l iff P is on l.

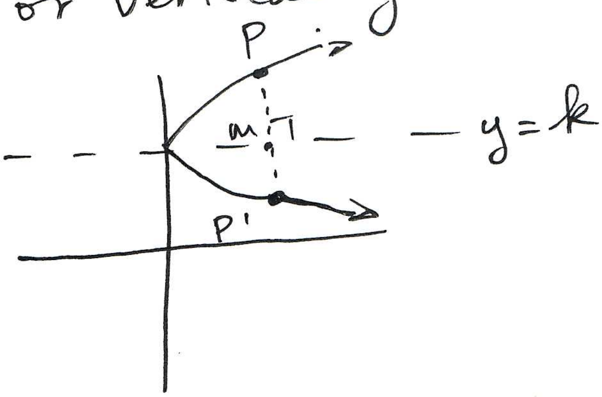
Ex.



P' & P are symmetric to the y-axis iff the y-axis is $\perp$ bisector of $\overline{PP'}$

$\overline{MP} \cong \overline{MP'}$ and $\overline{PP'} \perp$ y-axis at M

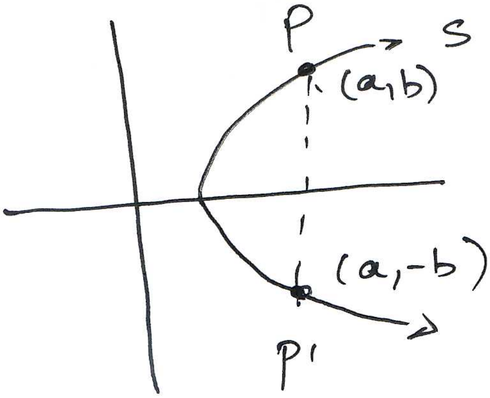
Symmetry to a line can be vertical or horizontal. It can be the x or y axis or shifted laterally or vertically.



Since $\overline{PP'} \perp y=k$ and $\overline{MP} \cong \overline{MP'}$ the relation is symmetric to the line $y=k$.

Symmetry with respect to the following lines:

x-axis If $(a,b) \in S$ where S is the set of points on the relation, then symmetry occurs if and only if $(a,-b) \in S$.

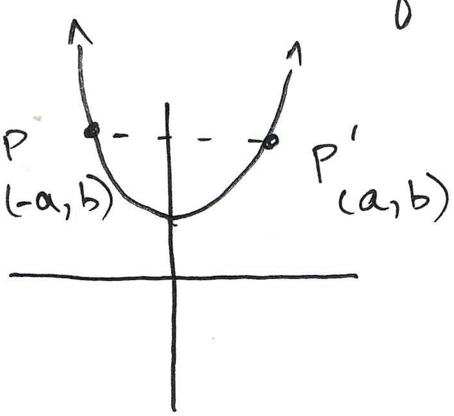


note $\overline{PP'}$ is $\perp$ to x-axis.

Functions symmetric to the x-axis are odd functions.

$$f(-x) = -f(x)$$

y-axis If $(a,b) \in S$ where S is the set of points on the relation, then symmetry occurs if and only if $(-a, b) \in S$



note $\overline{PP'}$ is $\perp$ to y-axis.

Functions symmetric to the y-axis are even functions.

$$f(-x) = f(x)$$

$y = x$ If $(a,b) \in S$ where S is the set of points on the relation, then symmetry occurs if and only if $(b,a) \in S$

ex. $xy = 6$

x	y
2	3
3	2
6	1
1	6

If $(2,3) \in S$ then $(3,2) \in S$.

If $(-2,-3) \in S$ then $(-3,-2) \in S$

$y = -x$ If $(a,b) \in S$ where S is the set of points on the relation, then symmetry occurs if and only if $(-b,-a) \in S$

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ex. $xy = -2$

x	y
2	-1
1	-2

If $a = 2$ $b = -1$ then $-b = 1$ $-a = -2$ therefore If $(2,-1)$ then $(-2,1)$.

Test 1 $xy = -2$ is symmetric to $y = -x$

Test 2 If $xy = -2$

If $a = 2$ and $b = -1$ then $a = -1$ and $b = 2$. True conconclusion $(a,b) \in S$ & $(b,a) \in S$

$xy = -2$ is symmetric to $y = x$

Test #3 If $xy = -2$

If $a = 2$ and $b = -1$ then $a = 2$ and $b = 1$. $(-2)(-1) \neq (2)(-1)$

$xy = -2$ is not symmetric to the y-axis.

Test #4 If $xy = -2$

If $a = 2$ and $b = -1$ then $a = 2$ and $b = 1$ $(2)(-1) \neq (2)(1)$

$xy = -2$ is not symmetric to the x-axis.