11-6 Natural Logarithms

Logarithms that are based on the number e rather than 10, we are using natural logarithms.
Natural logarithms are abbreviated $\ln x$.

$$\log_e x = \ln x$$

If $\ln e = x$ and $e^x = e$, then $x = 1$ and so $\underline{\ln e = 1}$

Each of the properties of common logarithms are also true for natural logarithms.

Product $\log_e mn = \log_e m + \log_e n$; $\ln mn = \ln m + \ln n$
Quotient $\log_e \frac{m}{n} = \log_e m - \log_e n$; $\ln \frac{m}{n} = \ln m - \ln n$
Power $\log_e m^p = p \log_e m$; $\ln m^p = p \ln m$
Equality $\log_e m = \log_e n$ then $m = n$; $\ln m = \ln n$; $m = n$

As with common logarithms, natural logarithms also can be reversed with an antilogarithm that is written as anti $\ln x$. If $\ln x = a$ then $x = \text{anti} \ln a$.
Normally anti $\ln x$ is written as e^x.

Example Problems

Convert $\log_6 254$ to a natural logarithm and evaluate.

$$\log_6 254 = \frac{\log_e 254}{\log_e 6} = \frac{\ln 254}{\ln 6} = \frac{5.5373}{1.7918} = 3.0904$$

Solve $6.5 = -16.25 \ln x$

$$\frac{6.5}{-16.25} = \ln x$$

$$-0.4 = \ln x$$

$$\text{antiln}(-0.4) = x$$

$$0.67 = x$$

Solve $3^{2x} = 7^{x-1}$

$$\ln 3^{2x} = \ln 7^{x-1}$$

$$(2x) \ln 3 = (x-1) \ln 7$$

$$(2x)(1.0986) = (x-1)(1.9459)$$

$$(2.1972)(x) = (1.9459)(x) - (1.9459)$$

$$0.2513(x) = -1.9459$$

$$x = -7.7433$$

30. $\log_{12} 56 \rightarrow \frac{\ln 56}{\ln 12} = \frac{4.02535}{2.48491} = 1.6199$

36. $6^x = 72 \rightarrow \ln 6^x = \ln 72 \rightarrow x \ln 6 = \ln 72$

$x = \frac{\ln 72}{\ln 6} = \frac{4.2766}{1.7918} = 2.3868$

42. $6.2e^{0.64t} = 3e^{t+1}$

$\ln 6.2 + 0.64t \ln e = \ln 3 + (t+1) \ln e$

$\ln 6.2 - \ln 3 - \ln e = t \ln e - 0.64t \ln e$

$\ln 6.2 - \ln 3 - 1 = t - 0.64t = 0.36t$

$\frac{\ln 6.2 - \ln 3 - 1}{0.36} = t$

$0.7613 = t$