11-4 Logarithmic Functions

The logarithmic function $y = \log_a x$ where $a > 0$ and $a \neq 1$ is the inverse of the exponential function $y = a^x$. Therefore, $y = \log_a x$ if and only if $x = a^y$

Example $10^2 = 100$ then $\log_{10} 100 = 2$

$2^5 = 32$ then $\log_2 32 = 5$

Properties of Logarithms

Product $\log_b mn = \log_b m + \log_b n$

Quotient $\log_b \frac{m}{n} = \log_b m - \log_b n$

Power $\log_b m^p = p \log_b m$

Equality $\log_b m = \log_b n$ then $m = n$

Base Changing Formula $\log_a C = \dfrac{\log_b C}{\log_b a}$

A Proof: Let $x = \log_a C$

$a^x = C$

$\log_b a^x = \log_b C$

$x \log_b a = \log_b C$

$x = \dfrac{\log_b C}{\log_b a}$

$\log_a C = \dfrac{\log_b C}{\log_b a}$

Sample Problems

20. $\log_{27} 3 = \frac{1}{3}$ then $\boxed{27^{\frac{1}{3}} = 3}$

28. $\left(\frac{1}{8}\right)^{-3} = 512$ then $\boxed{\log_{\frac{1}{8}} 512 = -3}$

38. $\log_8 16 = x$; $8^x = 16 \rightarrow 2^{3x} = 2^4 \rightarrow 3x = 4 \rightarrow \boxed{x = \frac{4}{3}}$

44. $\log_8 48 - \log_8 w = \log_8 6$

$\log_8 48 - \log_8 6 = \log_8 w$

$\log_8 \frac{48}{6} = \log_8 w$

$\boxed{8 = w}$

50. $2\log_5 (x-2) = \log_5 36$

$\log_5 (x-2)^2 = \log_5 36$

$(x-2)^2 = 36$

$(x-2)^2 = 6^2$

$x - 2 = 6$

$\boxed{x = 8}$

Evaluate $\dfrac{\log_7 9}{\log_7 3} = \log_3 9 = y \rightarrow 3^y = 9 \rightarrow 3^y = 3^2$

$\boxed{y = 2}$

Write $\log_{10} 5$ in terms of base e

$$\boxed{\log_{10} 5 = \frac{\log_e 5}{\log_e 10}}$$