The Number e

Comparing $y = 2^x$ with $y = 3^x$ we see that with the increased value for b, from 2 to 3, we get a steeper graph. Note that both graphs still contain the point (0, 1) as $1 = 2^0$ and $1 = 3^0$.
As the graph changes in its steepness, ~~as~~ we can draw a tangent line passing through (0, 1) and calculate the slope of the tangent line.

By inspection of $y = b^x$ on graphs determining a second point on the tangent line and calculating the value of m.

b	m
0.5	−0.69
1	0
2	0.69
3	1.1
4	1.4

Note that between b = 2 and b = 3, m = 1.0
This value for the base where the slope is equal to 1 is called e. $e \approx 2.718$

e is an irrational number just like π.

$f(x) = e^x$ is an exponential curve that includes the point of (0, 1) with a slope of 1 at that point.

The value of e is determined with an infinite series:

$$e = 1 + \frac{1}{1} + \frac{1}{1\cdot 2} + \frac{1}{1\cdot 2\cdot 3} + \frac{1}{1\cdot 2\cdot 3\cdot 4} + \cdots \frac{1}{1\cdot 2\cdot 3\cdot 4\cdots}$$

which leads to

$$e = 1 + 1 + \frac{1}{2} + \frac{1}{6} + \frac{1}{24} + \cdots \to 0$$

$$\underline{e \approx 2.718}$$

Exponential Growth / Decay in terms of e

$N = N_0 e^{kt}$

$N \to$ final amount
$N_0 \to$ initial amount
$k \to$ constant rate
$t \to$ time

* Notice the difference in the equation: $N = N_0(1+r)^t$

$(1+r) \approx e^k$

As k gets smaller then the expression $(1+r) = e^k$ becomes more accurate.

For continuously compounded interest our equation expressed in terms of e is $A = Pe^{rt}$

$A \to$ final amount
$P \to$ initial investment
$r \to$ annual interest rate
$t \to$ # years of investment

* Notice the difference in the equation: $A = P(1+\frac{r}{n})^{nt}$

$e^r \to (1+\frac{r}{n})^n$

Compare: $500 at 5% compounded semiannually for 10 years

$500 at 5% compounded continuously for 10 years

$A = P\left(1+\frac{r}{n}\right)^{nt}$

$A = 500\left(1+\frac{.05}{2}\right)^{2(10)}$

$A = \$819.308$

$A = Pe^{rt}$

$A = 500e^{(.05)(10)}$

$A = \$824.36$

Sample problem #9

$y = ae^{-kt} + C$

a = stemperature
k = constant
c = temperature of medium surrounding the cooling object
t = elapsed time of cooling

$T_0 = 160°F$
$C = 76°F$
$k = 0.23$
$t = 15$ minutes
$a = 160 - 76 = 84°$

$y = 84(e)^{-(0.23)(15)} + 76°F$

(a) $\boxed{y = 78.7°F}$

(b) $T_0 = 170°F$
$C = 72°F$
$k = 0.34$
$t = 5$ minutes
$a = 170 - 72 = 98°$

$y = 98(e)^{-(0.34)(5)} + 72$

$\boxed{y = 89.9°F}$

Too Cold because $89.9° < 135°$

#12 $y = 525(1 - e^{-0.038t})$

$y \rightarrow$ amount of population that received information in period t hours

t = # hours

a) $y = 525(1 - e^{-0.038(24)})$; $y = 314$ people

b) 90% of people = $.90 \times 525 = 473$ people $= y$

$473 = 525(1 - e^{-0.038t})$

$-\left(\frac{473}{525} - 1\right) = e^{-0.038t}$

$0.0990476 = e^{-0.038t}$

$0.0990476 = (e^{-0.038})^t$

$0.0990476 = 0.962713^t$

* $\log 0.0990476 = t(\log 0.962713)$

$\frac{\log 0.0990476}{\log 0.962713} = t$

$\boxed{60.8 \text{ hrs} = t}$

* Graph comparison results in an answer of approx. 61 hrs.

Log formula used will be shown in the next section.