Exponential Functions 11-2

$f(x) = b^x$

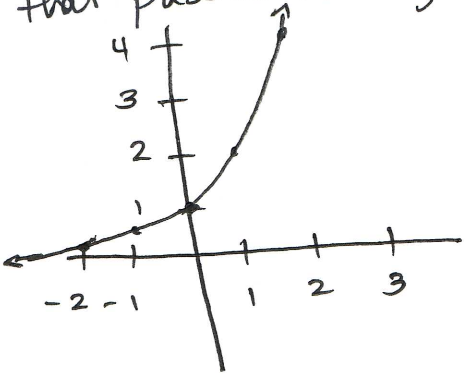
$x \to$ Variable exponent
$b \to$ a constant

If $b>0$ then b^x is defined for all rational values of x

If $b>0$ and x is irrational, then $b^x \approx b^r$ where r is a rational number obtained by rounding off x to a finite number of decimal places.

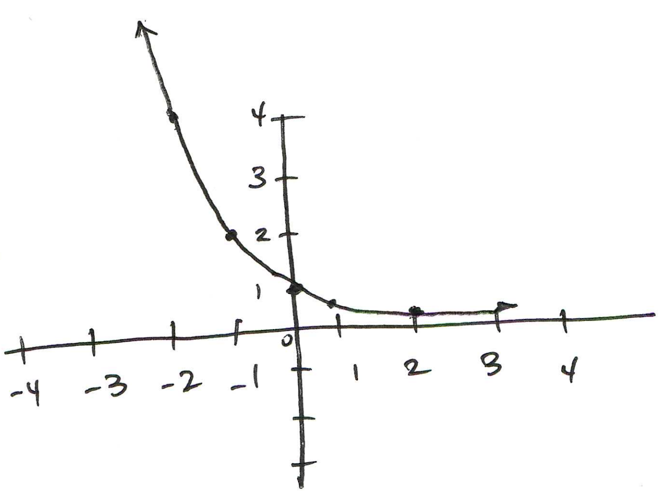
Ex. $b^{\pi} \approx b^{3.14}$

The graph of b^x where b is a positive constant is a curve that passes through the point $(0,1)$. $b^0 = 1$ $x=0, y=1$



x	y
-2	.25
-1	.05
0	1
1	2
2	4

$y = 2^x$



x	y
-2	4
-1	2
0	1
1	1/2
2	1/4

$y = \frac{1}{2}^x$

Characteristics of graphs of $y = b^x$

	$b > 1$	$0 < b < 1$
Domain	$\mathbb{R}$	$\mathbb{R}$
Range	$\mathbb{R} > 0$	$\mathbb{R} > 0$
Y-intercept	$(0, 1)$	$(0, 1)$
behavior	continuous, one to one increasing	continuous, one to one decreasing
horizontal asymptote	$(-)$ x-axis	$(+)$ x-axis
vertical asymptote	none	none

Generally $y = b^x + c$ the y-intercept will be $(0, 1+$

$y = 2^x + 3$ $\quad (0, 1+3) \rightarrow (0, 4)$

$y = 2^x - 3$ $\quad (0, 1-3) \rightarrow (0, -2)$

$c \rightarrow$ shifts the intercept vertically in the amount of $(1+c$

exponential decay - when a value (b^x) decreases in value over time. (decreasing curve)

exponential growth - when a value (b^x) increases in value over time - (increasing curve)

Equation that is used to determine the rate of decay or growth is: $N = N_0 (1+r)^t$

$N \rightarrow$ final number
$N_0 \rightarrow$ initial number
$r \rightarrow$ rate of growth or decay
$t \rightarrow$ number of time periods

Example: A mound of ants with 100 ants that grows by 5 ants per day for 30 days. What is the population after the 30 days?

$\frac{5 \text{ ants}}{100} = .05\%$

$N = 100(1+.05)^{30} = 432$ ants

Modify the Exponential Growth/Decay equation for accounting.

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

Compound interest formula

$A \to$ final amount
$P \to$ initial investement
$r \to$ annual interest rate
$n \to$ # of times interest is paid per year
$t \to$ # of years of investment

\$ 100 invested at 3.5% compounded monthly for 5 years

$P \to \$100$
$r \to .035$
$n \to 12$
$t \to 5$

$$A = 100\left(1 + \frac{.035}{12}\right)^{12(5)}$$

$A = \$119.09$

change $t \to 20$ years

$$A = 100\left(1 + \frac{.035}{12}\right)^{12(20)}$$

$A = 201^{.17}$

change $r \to 5.0\%$

$$A = 100\left(1 + \frac{.05}{12}\right)^{12(20)}$$

$A = 271.26$

change $P \to \$1000$

$$A = 1000\left(1 + \frac{.035}{12}\right)^{12(20)}$$

$A = \$2011.70$

$$A = 1000\left(1 + \frac{.05}{12}\right)^{12(20)}$$

$A = 2712.64$

change compounded daily

$$A = 1000\left(1 + \frac{.035}{365}\right)^{365(20)}$$

$A = \$2013.69$

$$A = 1000\left(1 + \frac{.05}{365}\right)^{365(20)}$$

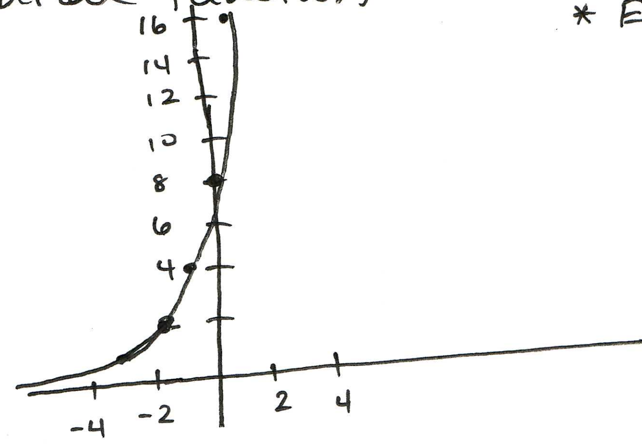
$A = 2718.09$

Practice Problems 11-2

Graph each exponential function

13. $y = 2^{x+3}$

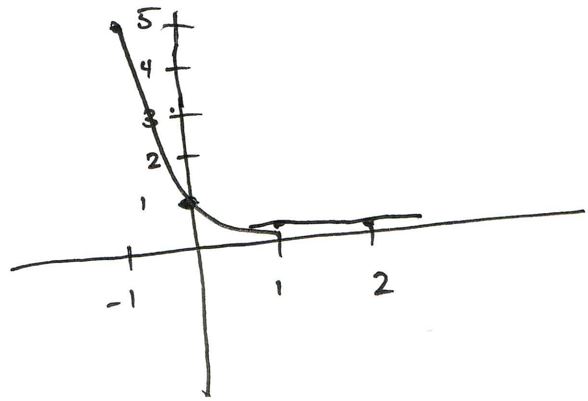
x	y
-3	1
-2	2
-1	4
0	8
1	16



* Exponential growth Model

16. $y = \left(\frac{1}{5}\right)^x$

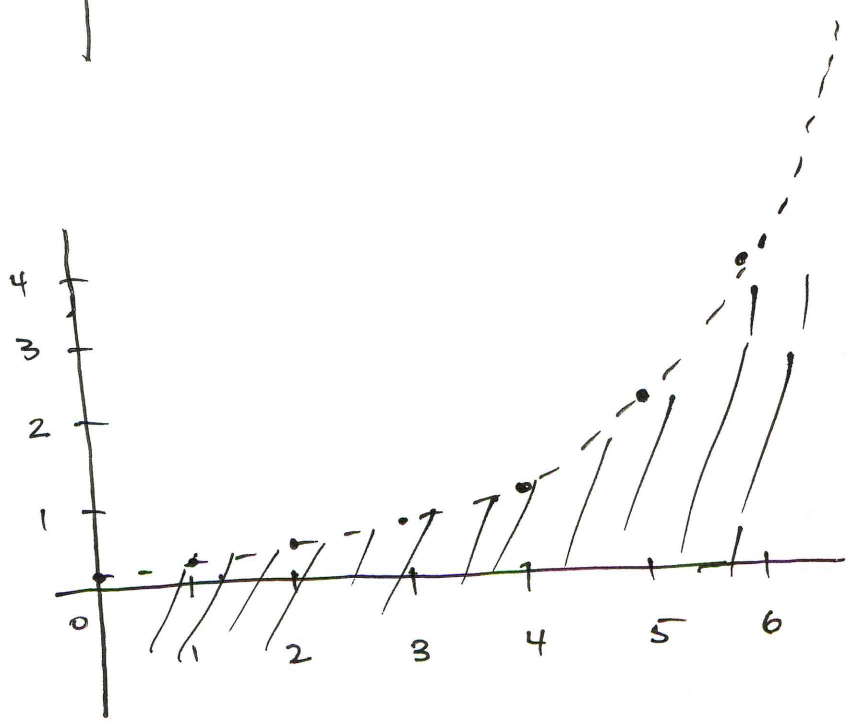
x	y
-1	5
0	1
1	$\frac{1}{5}$
2	$\frac{1}{25}$



* Exponential decay Model

18. $y < 2^{x-4}$

x	y
0	1/16
1	1/8
2	1/4
3	1/2
4	1
5	2
6	4



28. $P_n = P\left[\frac{1-(1+i)^{-n}}{i}\right]$

$P_n \rightarrow$ Present value
$P \rightarrow$ Total of payments
$n \rightarrow$ # periods
$i \rightarrow$ interest rate for period

$P_n = \$121{,}000$

annual interest $= .075$; $\frac{.075}{12} = .00625$ monthly interest

$n = 30$ years $= 360$ months

$\$121{,}000 = P\left[\frac{1-(1+.00625)^{-360}}{.00625}\right]$

(a) $\$121{,}000 = P(143.0176) \rightarrow \boxed{P = \$846^{\underline{05}}}$

(b) $n = 20$ years $= 240$ months

annual interest $= .0725$; $\frac{.0725}{12} = .00604$

$\$121{,}000 = P\left[\frac{1-(1+.00604)^{-240}}{.00604}\right]$

$\$121{,}000 = P(126.5415) \rightarrow \boxed{P = \$956^{\underline{21}}}$

(c) Total interest $= (240)(956^{\underline{21}}) - 121{,}000 = \$108{,}490.00$

$(360)(846^{\underline{05}}) - 121{,}000 = \$183{,}578.00$

(d) option A allows for smaller payments per month
option B allows for less interest paid over life of the loan