11-1 Real Exponents

Exponential Definitions

For any real number b, and a positive integer exponent n,

(a) If $n=1$, $b^n = b$
(b) If $n>1$, $b^n = b \cdot b \cdot b \cdot b \cdots$ n factors
(c) If $b \neq 0$, $b^0 = 1$
(d) If $b \neq 0$, $b^{-n} = \frac{1}{b^n}$

Exponential Properties

Given that m & n are positive integers and a & b are real numbers, then the following properties are true.

Product $a^m \cdot a^n = a^{m+n}$

Power of a Power $(a^m)^n = a^{mn}$

Power of a Quotient $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$ where $b \neq 0$

Power of a Product $(ab)^m = a^m b^m$

Quotient $\frac{a^m}{a^n} = a^{m-n}$ where $a \neq 0$

Definition of $b^{\frac{1}{n}}$ - For any real number $b \geq 0$ and any integer $n>1$, $b^{\frac{1}{n}} = \sqrt[n]{b}$

This also is true when $b<0$ and n is odd.

$$b^{\frac{m}{n}} = \left(b^{\frac{1}{n}}\right)^m \quad \text{and} \quad b^{\frac{m}{n}} = \left(b^m\right)^{\frac{1}{n}}$$

$$\sqrt[n]{b^m} \qquad\qquad \sqrt[n]{b^m}$$

Rational Exponents

For any nonzero number b, and any integers m and n with $n > 1$, and m and n have no common factors

$$b^{\frac{m}{n}} = \sqrt[n]{b^m} = \left(\sqrt[n]{b}\right)^m$$

except where $\sqrt[n]{b}$ is not a real number.

Irrational Exponents

If x is an irrational number and $b > 0$, then b^x is the real number between b^{x_1} and b^{x_2} for all possible choices of rational numbers x_1 and x_2 such that

$x_1 < x < x_2$

Ex: $2^{\pi} = \underline{\quad ? \quad}$

$\pi = 3.14$
Let $x_1 = 3.1$
Let $x_2 = 3.2$

$2^{\pi} = 8.825$
$2^{3.1} = 8.574$
$2^{3.2} = 9.190$

As a matter of estimation 2^{π} lies close to 2^3 and does not exceed 2^4, so 2^{π} is between 8 and 16 and can be interpolated near a value of 9.

11-1 Practice Problems

a) $5^{-4} = \frac{1}{5^4} = \frac{1}{625}$

b) $216^{\frac{1}{3}} = (6^3)^{\frac{1}{3}} = 6$

c) $\sqrt{m^3n^2} \cdot \sqrt{m^4n^5} = \sqrt{m^7n^7} = (mn)^{7/2}$ or $m^3n^3\sqrt{mn}$

d) $\sqrt{169x^5} = 13x^{\frac{5}{2}}$

e) $\sqrt[4]{a^2b^3c^4d^5} = |a|^{\frac{1}{2}}|b|^{\frac{3}{4}}|c||d|$

f) $\frac{(3^7)(9^4)}{\sqrt{27^6}} = \frac{(3^7)(3^8)}{(27)^{6/2}} = \frac{3^{15}}{(27)^3} = \frac{3^{15}}{3^9} = 3^6 = 729$

Express as a radical.

$$\frac{2^{\frac{2}{3}}}{2^{\frac{1}{3}}} = 2^{\frac{2}{3}-\frac{1}{3}} = 2^{\frac{1}{3}} = \sqrt[3]{2^1}$$

Simplify each expression.

$$\sqrt{d^3e^2t^2} = d \cdot |e| \cdot |t| \cdot \sqrt{d}$$

Solve each equation.

$724 = 15a^{\frac{5}{2}} + 12$

$712 = 15a^{\frac{5}{2}}$

$\frac{712}{15} = a^{5/2}$

$\left(\frac{712}{15}\right)^{\frac{2}{5}} = \left(a^{\frac{5}{2}}\right)^{\frac{2}{5}}$

$\left(\frac{712}{15}\right)^{\frac{2}{5}} = a$

$4.68 = a$

Find the solutions for $32^{(x^2+4x)} = 16^{(x^2+4x+3)}$

$$2^{5(x^2+4x)} = 2^{4(x^2+4x+3)}$$

$$5x^2+20x = 4x^2+16x+12$$

$$x^2+4x-12 = 0$$

$$(x+6)(x-2) = 0$$

$$x = -6, 2$$

Find the solutions for $9^{(x^2-4x+5)} = 3^{x^2+2x-11}$

$$3^{2(x^2-4x+5)} = 3^{x^2+2x-11}$$

$$2x^2-8x+10 = x^2+2x-11$$

$$x^2-10x+21 = 0$$

$$(x-7)(x-3) = 0$$

$$x = 7, 3$$